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SUBJECT EFFECTS IN CROSS MODALITY MATCHING

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "Subject Effects in Cross Modality Matching", submitted by Robert Paul Markley in partial fulfilment of the requirements for the degree of Master of Science.

Abstract

This experiment is a study of subject differences in psychophysical scaling and cross modality matching performance. It has been reported (Jones & Marcus, 1961) that the exponent of the psychophysical power function obtained by the method of magnitude estimation could be expressed as the product of components particular to a stimulus modality and to an individual making the psychophysical judgment. The subject component was interpreted as being due to either individual differences in perception of the stimulus or differences in Ss' use of numbers. (Ss emit numerical responses in a magnitude estimation task.) This paper provides further information on subject differences and investigates the accuracy of individual cross modality matches.

Individual psychophysical functions were obtained from 24 Ss for each of five tasks: (a) magnitude estimation of circle size, (b) magnitude estimation of numerosity, (c) magnitude production of force of handgrip, (d) cross modality matching of force of handgrip to circle size, and (e) cross modality matching of force of handgrip to numerosity. A no-prescribed-modulus procedure was used in condition (a). In condition (b) Ss were instructed to report the actual number of objects present. Handgrip responses in conditions (c), (d), and (e) were recorded on a Brush Mark II recorder.

Seven circles and seven random patterns of dots were prepared on 35 mm slides and projected on a screen in a darkened room. Exposure time was one second. In each condition Ss responded to a series of seven stimuli eight times. A different order of presentation was used each time the series was presented and the order of conditions was counter-balanced within each S.

Individual and group psychophysical functions were obtained for each condition. The data analyzed were the exponents of the resulting power functions.

Correlation coefficients were computed between each of the five conditions on individual exponents. High correlations were obtained between handgrip response conditions and low correlations were obtained between conditions with the same stimuli but different response procedures. The results were interpreted as indicating that individual differences in exponents reflected response rather than perceptual differences. Evidence is reported indicating that the Ss' responses in conditions (a) and (b) reflect two different response languages although in both conditions Ss emitted numbers.

The correlation between predicted and obtained individual exponents for cross modality matching was .49 for handgrip to numerosity and .57 for handgrip to circle size. The individual cross modality exponents did not support the multiplicative components hypothesis. The predicted cross modality exponents based on group functions were 0.99 for handgrip to circle size and 0.72 for handgrip to numerosity. The obtained exponents were 0.84 and 0.87, respectively.

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INTRODUCTION

The development of psychophysics and psychophysical methods by Fechner in the 1860's is well known to any student of the history of Psychology. His purpose was to obtain a measure of pure "subjective sensation" and relate this to a physical measure of a stimulus. Current interest in psychophysical scaling can be traced to the introduction of direct ratio scaling procedures by S. S. Stevens (1957) and the resultant set of empirical paradoxes and theoretical conflicts with the older indirect scaling procedures (Thurstone, 1927; Fechner, see Titchener, 1905; Edwards, 1957). Several historical summaries of this conflict are available (S. S. Stevens, 1957, 1961a, 1962; Boring, 1950; Torgerson, 1958). A neglected aspect of the controversy has been the question of the nature and sources of individual differences in psychophysical functions.

The Psychophysical Law

The application of an indirect scaling procedure usually results in the logarithmic psychophysical law

$$\psi = \underline{k} \log \phi \, . \qquad (1)$$

Equation 1 indicates that the value of a response (ψ) to a physical stimulus (ϕ) under a particular set of instructions to make a judgment is a function of the log of the physical value of the stimulus. The parameter $\underline{k}$ is an arbitrary unit of measurement constant. Thurstone's (1927) Law of Comparative Judgment and its

descendants (Torgerson, 1958; Sjöberg, 1962, 1963; and Eisler, 1963a) are modern representatives of this form of psychophysical scaling.

Thurstone's conceptual modification extended scaling procedures beyond the realm of purely physical stimuli. Any verbally identified attribute of a stimulus which Ss can recognize as being more or less present in members of a stimulus set can be scaled (e.g., political importance of Swedish monarchs, Ekman & Künnapas, 1963a).

Ultimately, the scaling procedures in the Thurstone-Fechner tradition require the S to respond on the basis of a perceived difference (or lack of difference) between stimuli. These are called indirect methods in that the S does not tell how much of the stimulus attribute is present, but merely whether there is more of it present in one stimulus than another.

In contrast, the procedures introduced by S. S. Stevens (1957) and modified by Ekman (1958, 1963a) ask the S to make a numerical estimate or in some other manner indicate a relative value of a variable present in the stimulus. These procedures result in the functional relationship:

$$\psi = k \phi^b \qquad (2)$$

which indicates that the value of the response (ψ) is a function of the physical measure (ϕ) of the stimulus taken to a power b. The constant k is again arbitrary while b is said to be characteristic of the stimuli.

The S's response (ψ), usually a number, is accepted as an immediate indication of the subjective scale value (sensation) of the stimulus. Thus Stevens' methods are referred to as direct scaling procedures. The resulting scales are said to be ratio scales since the Ss use numerical ratios in making responses. The basic assumption is that a S's numerical responses are linearly related to his subjective impressions or sensations.

The Power Law

The basic observation in direct scaling data is that equal stimulus ratios produce equal response ratios. (S. S. Stevens refers to "equal sensation ratios." See S. S. Stevens, 1962.) That is, given stimuli ϕ_1, ϕ_2, ϕ_3, ϕ_4, and responses to these stimuli ψ_1, ψ_2, ψ_3, and ψ_4 respectively, and if ϕ_1/ϕ_2 equals ϕ_3/ϕ_4, then ψ_1/ψ_2 will equal ψ_3/ψ_4. (Although ϕ_1/ϕ_2 is not necessarily equal to ψ_1/ψ_2.) The resultant expression for response as a function of stimulus is the psychophysical power law,

$$\psi = \underline{k}\,\phi^{b}$$

as in equation 2.

Stevens (1961a, 1962) modifies this function to

$$\psi = k\,(\phi - \phi_0)^b \qquad (3)$$

where ϕ_0 is an "effective threshold that obtains under the circumstances of the experiment" (S. S. Stevens, 1962, p. 30). All other terms are as in equation 2.

The added constant was introduced to "rectify the log-log plot of the magnitude function" (S. S. Stevens, 1961b). That is, the data did not fit a power function without the addition of ϕ_0. It was suggested that ϕ_0 is an absolute threshold parameter. Ekman (1959) found a threshold concept inappropriate and considered ϕ_0 to be merely an additive constant which may be positive or negative. Corso's (1963) review of the threshold concept further explicated this point.

The exponent b varies with the stimulus modality (S. S. Stevens, 1957; Stevens and Galanter, 1957). It may also be affected by adaptation and contrast (S. S. Stevens, 1962), range of stimuli (Björkman & Strangert, 1960; Strangert, 1961; Ekman, Frankenhäuser, Levander, & Mellis, 1964), size of modulus (Wong, 1963), and location of standard in series (Poulton and Simmonds, 1963). The influence held by these additional variables may interact with the type of direct scaling procedure used to obtain judgments.

Scaling Procedures.--S. S. Stevens (1957) identifies four types of direct procedures for obtaining ratio scales: ratio

estimation, ratio production, magnitude estimation, and magnitude production.

In ratio production the S adjusts a variable to a standard stimulus to produce a sensation ratio specified by E. This may be either a multiple of the standard or a fraction of the standard (fractionation technique). For reasons yet unknown, introduction of a second standard (bisection procedure) results in a scale non-linear with fractionation scales. A variant of ratio production similar to the method of constant stimuli, has the E setting the two stimuli and then asking the S if the stimulus ratio is equal to, greater than, or less than a prescribed ratio. The validity of this procedure has been challenged by Garner (1954, 1959). Furthermore, Michels and Helson (1954) derived ratio production results from a conjunction of adaptation-level theory and Fechner's Law.

Ratio estimation is the procedural inverse of ratio production. The S is presented with a pair of stimuli and supplies a numerical estimate of the ratio of the two. Various constraints and modifications in this method have been used. A variant developed by Ekman has been the primary procedure used in an extensive series of studies at the University of Stockholm (Ekman, 1958, 1959, 1961, 1962, 1963a; Eisler, 1960, 1962a, 1962b, 1962c, 1963b, 1963c, 1963d, 1963e; Björkman & Strangert, 1961; Strangert, 1961; Engen & Lindström, 1963; Künnapas, 1960, 1964; Künnapas & Sillen, 1964; Ekman & Künnapas, 1962a, 1962b, 1963a, 1963b, 1963c).

Much of S. S. Stevens' early work used ratio estimation or production procedures (S. S. Stevens, 1955). More recently, Stevens has relied on a procedure he has called magnitude estimation (S. S. Stevens, 1956). In this method the E presents a stimulus and has the S assign a number to it. The S is instructed to assign numbers proportional to his subjective impressions of the stimuli. There are two varieties of this procedure: a no-prescribed-modulus procedure and a prescribed-modulus procedure. In the latter method the E presents a stimulus as a standard and assigns it a numeral (modulus). In the former the S is free to assign to the standard any number that he feels is appropriate.

Stevens has found that the prescribed and no-prescribed-modulus procedures yield similar results although one or the other may be more suitable for a particular modality (S. S. Stevens & Harris, 1962). Ss were thought to be more comfortable with the no-prescribed-modulus procedure. It was claimed that size of the modulus does not affect the psychophysical function (S. S. Stevens, 1956, 1962; J. C. Stevens, 1958). However in these studies (of loudness), the location of the standard within the stimulus series was confounded with different values of the modulus. Wong (1963) corrects this defect and finds that the modulus does affect exponents obtained from judgments of length of line.

In magnitude production, the procedure is the reverse of magnitude estimation. The E supplies a number and the S adjusts a stimulus to match the subjective value of the stimulus to the number.

The inadvertant impression given by workers in this field is that these four types of methods yield the same ratio scale when used in the same stimulus modality. In fact, they do all result in a psychophysical power function. But the parameters (principally the exponent) of the power function have not shown a high degree of invariance between procedures. Cross procedural studies carried out in the same laboratory at the same time seem to produce similar exponents (J. C. Stevens & Mack, 1959). Experiments reported from different laboratories at different times, using different scaling methods, have not resulted in consistant exponents. An example would be the several studies of subjective weight (S. S. Stevens & Galanter, 1957; Jones & Marcus, 1961; Eisler, 1960; Pradhan & Hoffman, 1963). Intra-procedural variance in exponents is reported by Strangert (1961) for ratio estimations of numerosity and by Poulton & Simmonds (1963) for magnitude estimations of subjective greyness. Sternbach & Tursky (1964) failed to replicate Stevens, Carton, & Shickman's (1958) exponent for subjective shock intensity.

Research into boundary conditions affecting the exponent is just now beginning to appear (Jones & Woskow, 1966).

Direct scaling procedures are distinct from absolute judgment procedures (Helson, 1964) in that the _S_'s responses are (in theory) continuous rather than categorical or discrete. The _S_ either emits a number or makes an adjustment of an effectively continuous physical response apparatus.

Scale Values and Exponents.--In the procedures of magnitude estimation and magnitude production, _S_'s responses are pooled to obtain a scale value. These responses, especially those of magnitude estimation, are usually skewed. Originally the median response was used as a scale value. Stevens (1962) and Jones & Marcus (1961) show that the geometric mean of the responses is the appropriate estimate of the scale value.

A power function becomes a linear equation when both variables are converted to logarithms. Thus Equation 2 becomes

$$\text{Log } \Psi = \underline{b} \text{ Log } \phi + \text{Log } \underline{k}. \qquad (4)$$

Hence, to obtain the psychophysical function, the responses and stimulus values must be transformed to logarithms, a mean log response (log scale value) for each stimulus obtained, and the log scale values plotted against log stimulus values. If a straight line gives an adequate fit to the data points, then the power function is an adequate first order description of the data. The parameters of the straight line become the parameters of the psychophysical power function (Lewis, 1960).

Cross Modality Matching

The most immediate criticism of direct scaling procedures is that these rely on the assumption that the S's use of numerals is like the bookkeeper's use of numerals. Garner (1954) first suggested that this might not be the case. Attneave (1962 [written before 1959]) suggested that the S may be matching the subjective magnitude of the stimulus with a subjective correlate of a number. Thus if a psychophysical function were available for the S's use of number, the conflicts between direct and indirect scaling might be resolved.

Stevens reacted to criticism of Ss' making numerical responses by devising response procedures that did not involve a numerical estimate. In these procedures the S adjusts a variable continuum (e.g., loudness of a sound) until it is subjectively equal to another continuum (e.g., the redness of a color patch).

Given Equation 2 for two different continua

$$\psi_1 = k\phi_1^{m}$$

$$\psi_2 = k\phi_2^{b} \qquad (2a)$$

where ψ_1 and ψ_2 are thought of as subjective magnitudes; if the S is instructed to set $\psi_1 = \psi_2$ and letting $k = 1$ then

$$\phi_1^{m} = \phi_2^{b}. \qquad (5)$$

Taking logarithms and rearranging

$$\log \phi_1 = \frac{b}{m} \log \phi_2$$

and finally

$$\phi_1 = \phi_2^{b/m}. \tag{6}$$

Equation 6 is the predicted result of cross modality matching (CMM) procedures (S. S. Stevens, 1959a). The expected exponent of the cross modality function is given by the ratio of the exponents _b_ and _m_. Stevens regards this as "a necessary _condition_ (but not sufficient condition) for the validity of the...subjective scales" (S. S. Stevens, 1959a, p. 202). It is not a sufficient condition since the cross modality procedures indicate only the relative forms of the magnitude functions in different sense modalities.

Similarly, accepting Attneave's suggestion that under magnitude estimation instructions the _S_ is matching a number to a stimulus, _and_ assuming that the power function is a "true" psychophysical law, one can take

$$\psi_{nu} = \phi_{nu}^{n}$$

$$\psi_1 = \phi_1^{m} \tag{2b}$$

$$\text{and} \qquad \psi_2 = \phi_2^{b}$$

where ψ_{nu} is a subjective magnitude of a number, ϕ_{nu} a number, and _1_ and _2_ are referents for two physical continua. Setting

$$\psi_{nu} = \psi_1$$

then $$\phi_{nu}^{\,n} = \phi_1^{\,m}$$

and $$\phi_{nu} = \phi_1^{\,m/n}. \tag{7}$$

Likewise if

$$\psi_{nu} = \psi_2$$

then $$\phi_{nu} = \phi_2^{\,b/n}. \tag{8}$$

Equations 7 and 8 represent experimentally obtained psychophysical functions for two continua. If $n = 1$, these are specific instances of the Psychophysical Law as interpreted by Stevens.

Now setting $\phi_{nu} = \phi_{nu}$ as in the CMM procedure then

$$\phi_1^{\,m/n} = \phi_2^{\,b/n}$$

and $$\phi_1 = \phi_2^{\,b/m} \tag{6}$$

Note that the n drops out in CMM and hence that its value is immaterial in so far as the empirical results of cross modality matching (CMM) are concerned. This result illustrates the insufficiency of CMM as a validation of the direct scaling procedures.

S. S. Stevens (1959a) introduced cross modality matching in a study of subjective impressions of loudness, mechanical vibration of a finger, and intensity of electric shock. The loudness function was taken from previous research. Magnitude

estimation scales of vibration and shock intensity were obtained as part of the experiment. From these three scales cross modality matching (CMM) predictions were made. Then six groups of ten Ss each made CMM judgments of Shock-to-Loudness, Loudness-to-Shock, Loudness-to-Vibration, etc. In all, 30 Ss were used, most of them taking part in more than one experimental session. The results confirmed the cross modality prediction both in the approximate size of the exponent and in the appearance of a slight curvature in the log-log plots of matches involving shock.

Stevens (1959b) confirmed the scale for finger vibration and found that vibration applied to the arm did not follow a power function (unless a healthy threshold correction was included). Group mean judgments of matching of finger vibration to arm vibration fell on a predicted curve. The predicted curve was constructed by plotting a given change in arm vibration with a change of the finger vibration necessary to produce a change in magnitude estimation equal to a change in magnitude estimation produced by the given change in arm vibration.

J. C. Stevens and J. Mack (1959) introduced scaling of force of handgrip. Ratio production, magnitude estimation, and magnitude production methods were used. A final combined estimate of the exponent was 1.7. A magnitude estimation procedure obtained a scale of force applied to the palm with an exponent of 1.1. CMM of exerted handgrip force to force applied to the palm yielded an exponent of 0.66, quite close to the predicted 0.65.

Handgrip was also matched to the apparent heaviness of lifted weights. The predicted exponent was 0.85. The results gave an exponent of 0.79. There was no mention made of the nature of the subjects used in these experiments or of the amount of experience they had in making psychophysical judgments.

Stevens, Mack, & Stevens (1960) added five more continua to the list of those measured by force of handgrip. These were shock intensity, brightness of white light, loudness of white noise (SPL), loudness of 1000 cycle tone (SPL), and mechanical vibration. Ten to twelve Ss made two to four CMM judgments of five or six stimuli in five conditions. A total of 26 Ss participated in varying amounts in the five experiments. In this study Ss did not make any single modality or numerical judgments of the various continua used or of force of handgrip. Single modality exponents were taken from previous experimentation. Predicted and obtained CMM exponents were 2.06 and 2.13 for shock, 0.20 and 0.21 for brightness, 0.35 and 0.41 for loudness of white noise, 0.35 and 0.35 for loudness of pure tone, 0.56 and 0.56 for vibration, respectively.

J. C. Stevens & S. S. Stevens (1960) obtained psychophysical functions for subjective warmth and cold. Cross modality matching judgments with force of handgrip were then obtained. The obtained CMM exponents were 0.96 and 0.60 for warmth and cold respectively. The predicted CMM exponents were 0.94 and 0.59.

[illegible]

[illegible] cotton plants. [illegible]

[illegible]

[illegible]

[illegible]

[illegible] (1963) [illegible]

[illegible] the list of [illegible]

[illegible]

[illegible] variation. [illegible]

[illegible]

[illegible] populations [illegible]

[illegible]

[illegible]

[illegible]

[illegible] respectively.

[illegible] (1961) [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

S. S. Stevens & Harris (1962) matched subjective roughness and smoothness (sandpaper stimuli) with the loudness of wide band noise. The CMM exponents were as predicted.

S. S. Stevens & Guirao (1963) scaled subjective length of lines. The magnitude estimation exponent was about 1.0. They then matched length to loudness and softness of a tone and to brightness and dimness of a light. The results were as predicted regarding both size and sign of the exponent, i.e., softness is the inverse of loudness.

Abbey (1962) obtained magnitude estimation scales for numerosity and pitch and cross modality matches of pitch to numerosity and of numerosity to pitch. Four different groups of five _S_s participated in each experiment. The numerosity exponent was 1.2. The pitch function was not linear in log-log plots. A predicted CMM curve was constructed in the same manner as Stevens (1959b). (See above.) The numerosity to pitch match fit the predicted curve. The reverse match failed although the curves were similar in shape. The failure was discussed in terms of difficulty in judgment of pitch.

Finally, and of incidental interest here, is the study by White (1964). She obtained subjective CMMs of light intensity, loudness of tone, and intensity of shock. Thirty different _S_s were then presented with shock intensities to establish their lower thresholds. Then a light and a tone were selected that were subjectively equal to a supra-threshold shock. During a

series of experimental conditions the three stimuli were presented to the Ss and eight autonomic physiological measures were taken. Reactions on six of the eight measures were equal for the three stimuli.

The results of the above CMM studies have conformed remarkably well to the predictions. S. S. Stevens (1961a) reports no errors in prediction of exponents greater than 0.07. Unfortunately it seems that the existence of the CMM phenomenon neither solves the direct-indirect scaling conflict nor establishes the superiority of the Power Law.

Further theoretical analysis has shown that the cross modality match is equally predictable from Fechner's Law. As has been developed by Treisman (1964) and Luce & Galanter (1963), if

$$\psi_1 = m \log \phi_1$$

$$\psi_2 = b \log \phi_2 \qquad (1a)$$

where ψ is again a subjective magnitude, then setting

$$\psi_1 = \psi_2$$

sets $$m \log \phi_1 = b \log \phi_2$$

and $$\log \phi_1 = b/m \log \phi_2$$

thus $$\phi_1 = \phi_2^{b/m} \qquad (6)$$

However, as Luce & Galanter (1963) point out, the m and b values in Equation 1a represent arbitrary choices of units and therefore b/m is not ordinarily predictable from the data of the Thurstonian scaling situation.

Ekman (1964a, 1964b) takes this sort of argument one step further back. He assumes that Fechner's law relates any stimulus variable, including numbers as a stimulus variable, and a subjective variable. Thus we have

$$\psi_{nu} = c + d \log \phi_{nu}$$

$$\psi_{m} = i + j \log \phi_{m} \tag{9}$$

where ψ is the subjective variable and ϕ the stimulus variable, nu signifies the number continuum, and m any other stimulus continua. The S then sets

$$\psi_{nu} = \psi_{m};$$

that is to say, he matches a stimulus ϕ_{nu} to a stimulus ϕ_{m} then

$$c + d \log \phi_{nu} = i + j \log \phi_{m}$$

$$\log \phi_{nu} = \frac{i - c}{d} + \frac{j}{d} \log \phi_{m}$$

now let $\frac{i - c}{d} = \log k$ and $\frac{j}{d} = b$

then $\log \phi_{nu} = \log k + b \log \phi_{m}$

and $$\phi_{nu} = k \phi_{m}^{b}. \tag{10}$$

Equation 10 is identical to equations 7 and 8. It indicates that the relation between two stimulus continua, one used as a response and one used as a stimulus, is a power function. A cross modality

prediction can easily follow. In light of this sort of model all direct scaling procedures are forms of cross modal matches with the usual numerical matching being a specific most commonly appearing instance.

Ekman (1964a, 1964b) can find no way to distinguish operationally between the two conceptions of the power law (i.e., between ψ as a response variable, and therefore another indirect indicator of sensation, and ψ as a direct measure of subjective magnitude or perceptual process).

This is essentially the question raised by Garner, Hake & Eriksen (1956) generally in regard to the concept of perception and specifically about use of such a concept in psychophysical scaling. (See also Attneave, 1962; Brown, 1961; Jones & Marcus, 1961.) Garner, Hake & Eriksen take the position that the use of a perceptual construct to describe or explain behavior requires not only that it be operationally defined but that it be defined by a set of orthogonal operations which converge upon the construct such that alternative descriptions and constructs are excluded.

The problem for psychophysical scaling is to find converging operations which will discriminate between a response or a perceptual construct's explanation of the results. If both constructs are ultimately necessary then the task is to partial out their effects and interactions.

A similar problem arises in interpretation of individual differences in performance on psychophysical scaling tasks. Such differences may be thought of as reflecting differences in handling of either the stimulus input process or of the response media.

Individual Effects

The studies of psychophysical scaling reported so far have paid little attention to the individual subject. Supra-threshold psychophysics has been interested primarily in establishment of nomothetic rather than ideographic laws. With the few exceptions to be reported below, inter-individual variation has been treated as either a nuisance or an error factor. Recently intra-individual variability of responses has become a parameter in a series of Swedish studies on the relationships between direct and indirect scales (Ekman, 1962; Ekman & Künnapas, 1962b, 1963a, 1963b; Eisler, 1962a, 1962b, 1963a, 1963b, 1963d, 1963e). With the exception of a few studies relating pitch perception and musical abilities, no systematic studies of population differences in psychophysical functions have been performed.

Garner (1954, 1959), in his demonstration of context effects in judgments of half-loudness (see above), found highly reliable and consistent individual differences. He presented Ss with a standard tone of 90 db SPL, and then a series of comparison stimuli. Ss were to judge whether each comparison stimulus was greater or less than one-half as loud as the standard. Three non-overlapping

ranges of comparison stimuli were used. The group mean half-as-loud values were in each case at the midpoint of the range of comparison stimuli. Each S in a group made judgments of six stimuli 100 times each. Ss' individual half-loudness values were widely spread out over the range of comparison stimuli. The correlation of Ss' half-loudness values for the first 300 judgments vs. the last 300 judgments was 0.84. A consistency coefficient based on the number of judgments of greater than one-half in the first 100 trials vs. the number of such judgments in the last 100 trials was 0.77. Garner concluded that Ss have no real direct perception of what half-as-loud means but that they accept the context of the experiment. The Ss were said to make highly reliable judgments that lack any sort of validity.

Stevens, Mack, & Stevens (1960) suggested that individual variations in exponents of the power function may arise from differences in operating characteristics of "sensory transducers" and from "the simple fact of human variability." They present no data on the matter. (See also S. S. Stevens, 1961b.)

Beck & Shaw (1961) reported wide variability in individual magnitude estimation judgments of pitch. Additionally, there were differences between musical and non-musical groups of Ss. However, only group scale values and interquartile ranges of responses to each stimulus were reported.

J. C. Stevens & Mack (1959) obtained individual psychophysical functions for force of handgrip for two groups of ten Ss by the method of magnitude production. Members of one group made ten responses (squeezes) each to numerical stimuli. Another group made seven responses to the same numbers. The groups used different hand dynamometers. The median individual exponents were 2.0 and 1.9 respectively. Semi-interquartile ranges of individual exponents were 0.45 and 0.40. Surprisingly seven Ss took part in both groups. The authors reported that they were not very consistent from experiment to experiment. Plots of these 20 individual functions were published separately by S. S. Stevens (1962). They appear satisfactorily linear in log-log coordinates. Two other groups made magnitude estimation judgments of handgrip force. Median and composite exponents were around 1.7. The semi-interquartile ranges were 0.20 and 0.25. These plots were published in S. S. Stevens (1961b).

Pradhan & Hoffman (1963) obtained individual data for six Ss in a study of range and stimulus spacing on magnitude estimation of subjective weight. Ss judged nine weights 20 times in each of eight experimental sessions. Psychophysical functions were obtained for each S in each session. There were apparent Range X Spacing X Subject interactions but these were highly ambiguous and not interpreted.

No test of goodness of fit has been considered appropriate to test the linearity of regression of the log-log plots of

psychophysical data. To fulfill this function Pradhan & Hoffman devised a statistical test "based upon the technique of estimation of simultaneous confidence intervals for several means" (p. 536). (See Scheffé, 1953.) The log-log plots of scale values for five of the six Ss were interpreted as significantly deviating from linearity in most of the conditions. The group data, however, were adequately described by the power function.

The results were interpreted as indicating that individual psychophysical functions did not follow Stevens' Law but that averaging over Ss results in a power function. Only one plot of an individual's data for one condition was presented.

Ekman, Hosman, & Lindström (1964) report ten individual psychophysical functions for subjective roughness, smoothness, and "pleasantness" or "preference" of sandpaper stimuli. A ratio estimation procedure was used. Visual inspection of the log-log plots indicated that the power function adequately described the data. Roughness and smoothness exponents for each S were approximately equal to each other but opposite in sign. The preference function in nine of the ten cases was linear with the smoothness function. Inter-individual variation of exponents was large--a range from 0.8 to 3.5 for roughness and from -0.8 to -3.0 for smoothness.

This variation was discussed in terms of either (a) individual differences in sensitivity or (b) differences in use of response language.

The authors then went on to suggest that a high inter-modal correlation over Ss of a measure of the range of subjective response (the exponent) would indicate a response handling interpretation.

A study by Jones & Marcus (1961) has been the most extensive examination of individual effects so far in the literature. Forty-nine Ss made magnitude estimation judgments (prescribed-modulus procedure) of three modalities: weight, taste (saltiness), and smell (benzene). Seven stimuli were used in each modality, and each S judged each stimulus twice. Individual psychophysical functions of the form

$$\log \psi = \log a + b \log \phi \qquad (11)$$

were fitted to each of two orders (trials) for each modality for each S. The slope parameter b served as data for further analysis.

Median individual exponents and semi-interquartile ranges for the three modalities—weight, taste, and smell—were 1.7, 1.05, and 0.56, and 0.38, 0.36, and 0.23 respectively.

Analysis of variance performed on the 294 exponents yielded significant Subject, Modality, and Subject X Modality interaction effects. The modality effect was expected. The S effect was interpreted as implying "a consistency in the use of numerals by a given S from modality to modality." (p. 41)

Further analysis was carried out to determine the meaning of the Subject X Modality interaction. It was suggested that

the interaction represented a multiplicative effect of <u>S</u> on modality. That is, the exponent was the product of a constant stimulus component, characteristic of a particular modality, and of a component characteristic of an individual.

A multiplicative model of analysis of variance (Williams, 1952) was used (in a simplified form) to test the hypothesis of the multiplicative nature of the Subject X Modality interaction. Only the weight and taste exponents were used. The smell data was excluded due to a marked skewness. This reanalysis resulted in the elimination of the Subject X Modality interaction. This result was interpreted as support for the multiplicative hypothesis.

Jones & Marcus suggested that the form of the power function be revised to read

$$\psi = \underline{k}\,\phi^{bc_i} \tag{12}$$

for an individual where c_i is an individual component and b is characteristic of the stimulus. For group data the function becomes

$$\psi = k\,\phi^{b\bar{c}} \tag{13}$$

where $\bar{c}$ is an average of individual components. Whether this individual component was a response or a perceptual variable was undeterminable.

The foregoing survey has reviewed psychophysical scaling literature for CMM and individual results. S. S. Stevens and co-workers have provided a considerable amount of evidence for the predictability of group CMM functions. No individual CMM functions have been reported.

Studies of individual psychophysical functions were rare. These reports were contradictory in regards to the shape of individual functions. It was suggested that the exponent of the power function is a product of an average individual and a stimulus component. No definite specification of the meaning of differences in individual values of the psychophysical function's parameters could be made.

The present study originated in the findings of Jones & Marcus (1961) in regards to individual and stimulus contributions to the exponent of the psychophysical function. Individual functions were obtained in five direct scaling situations. These data provided further information on individual differences in psychophysical functions, the accuracy of individual CMM predictions, and the generality of the Jones & Marcus two component theory.

METHOD

Subjects

The Ss were twenty-four male undergraduate students enrolled in an introductory psychology course at the University of Alberta. Participation in psychological experiments was a required part of the course work.

Apparatus and Materials

Seven circles and seven patterns of dots were used as stimuli. The projected inside diameters of the circles were 0.5, 1.0, 2.25, 3.56, 4.69, 5.81, and 7.25 inches. The seven dot patterns contained 9, 14, 20, 27, 38, 58, and 81 dots. Dots within each pattern were located randomly with the restriction that all arrays should spread uniformly over approximately the same circular area.

The stimuli were drawn in black india ink on a blank white card. The cards were then photographed with color film and a 35mm slide made up out of the color negative. The circle and dot stimuli were projected onto a screen by a Bell & Howell Explorer 35mm slide projector located opposite the S. The projector lens was 54 inches from the screen. Exposure time was controlled by a manually operated Wollensak Aphax shutter mounted in a wooden panel four inches in front of the projector lens.

The stimuli were presented to the S for a period of one second each. A seven to ten second period of darkness between stimuli allowed the S to record his judgments.

To record force of handgrip the S grasped and squeezed a fixed handle. This apparatus was the aluminum frame and grip from a Pacific Medical Tensiometer. Tension on the cable was registered on a strain guage mounted as a wheatstone bridge in the frame. Output from the guage was amplified and recorded as a needle deflection on a Brush Mark II recorder.

The recorder was scaled so that a 1mm deflection of the recording needle represented five pounds force. The needle response was linear through its range of from zero to two-hundred pounds. Response record charts were read to the nearest 2.5 pounds of force. The E marked the chart when the S indicated the force exerted was appropriate to the stimulus.

The experiment was conducted in a 6'6" x 11'8" research room at the Department of Psychology, University of Alberta, Edmonton. A screen constructed out of plywood and black plastic sheeting divided the room. Set into this screen was 28 x 23 inch window covered by white construction paper. This window served as a projection screen.

The S was seated at a 24 inch square table facing the projection screen. The back of the S's chair was fixed against the end wall of the room 62 inches from the screen. The S adjusted the position of the table to obtain a comfortable grip on the handgrip apparatus.

On the table were a pencil, answer sheets, and a six volt light shielded so that the S could see the answer sheet yet

not have a bright light showing between him and the projection screen. The handgrip device was clamped to the table near its upper right hand corner.

Procedure

Estimates of subjective magnitude were obtained in five conditions. Size-of-circles and numerosity (the number of dots present) judgments were obtained via the method of magnitude estimation. The force exerted on a handgrip was estimated by a magnitude production procedure. Cross modality judgments were then obtained by matching force of handgrip to size of circles and numerosity. Each S took part in all five conditions.

Prior to the experiment the Ss were allowed to view the apparatus and were read a general statement about the nature of perceptual research. Ss were then seated and allowed to adjust the position of the table. Instructions were then given for the first of the single modality conditions.

For the circle size condition the instructions were as follows:

> Now I am going to present to you a series of circles. Your task is to judge the size of these circles by assigning numbers to them. Call the first circle any number that seems appropriate to you.
>
> Your task is to assign numbers to the remaining circles that are proportional to your subjective impressions. For example, if a circle is three times the size of the first, assign it a number three times as large as the first. If it seems one-fifth the size, assign a number one-fifth as large. If you are not sure guess. Record your estimates on the answer sheet. Are there any questions?

The *S* would then judge each of the seven circles four times. Four different random orders of stimuli were prepared. These were the same for all *Ss* and occurred in the same order for each *S*. That is, all *Ss* were presented with the same series of twenty-eight slides.

S would then make judgments in the next modality. For numerosity the instructions were:

> I am now going to present to you a series of dot patterns. I would like you to estimate the actual number of dots in each pattern. Make an estimate for each pattern. Mark your estimates on the answer sheet. If you are not sure guess. Are there any questions?

After four judgments of each stimulus the next condition would be undertaken.

For the force of handgrip condition the instructions were:

> Now, you are asked to begin by exerting a MODERATE squeeze on the handle. Let the number 10 stand for the apparent value of the force you exert. You will then be asked to exert forces in proportion to other numbers. For example, if given the number twenty-five you would attempt to exert an apparent force $2\frac{1}{2}$ times as great as the force called ten.
>
> You may adjust your grip until you feel that the appropriate force is reached. Then signal as precisely as possible the moment when this occurs. Then try to hold this force till I signal you to relax.
>
> Be sure to sit up and back in your chair. Since we are going to be making a fairly large number of judgments with this apparatus you may stop and rest whenever you feel particularly tired. Please tell me if you wish to rest. Are there any questions?

Numbers to be matched in this condition were 2, 3, 6, 10, 20, 30, and 40.

The three single modality conditions were undertaken in a counterbalanced ABCCBA order. A short rest period was given in the middle of the session. Thus the typical S would make 28 circle judgments, then 28 handgrip judgments, then 28 numerosity judgments, rest briefly and then repeat the series backwards--dots, handgrip, circles. The second series of 28 stimuli in any condition was the mirror or inverse of the first 28. The six possible ABCCBA type orderings were randomly arranged and each assigned to four Ss by a lottery method.

After completion of the single modality judgments, S was given a ten to 15 minute rest period. S was then returned to his seat and cross modality judgments were made. Cross modality instructions for the first series of 28 judgments (four for each stimulus) were:

> I am now going to again present to you a series of circles (dot patterns). You are to again estimate the size of the circle (number of dots). This time however you will make your estimations by squeezing the handle with a force that appears to match the apparent magnitude of the circle (number of dots). You may adjust back and forth till you reach the appropriate force. Tell me as precisely as possible the moment when the matching force is achieved. Then hold this force till I signal you to relax.

On the subsequent three series the S was instructed:

> Now we will judge a series of dot patterns (circles) again using the handgrip to estimate the magnitude of the image on the screen. Follow the same procedures as were previously followed.

Again, the split-half counterbalancing (ABBA) procedure was followed. Ss were assigned randomly to one of the four possible

arrangements of conditions (ABBA, BAAB, ABAB, & BABA) independent of the single modality arrangements.

Stimulus orders and arrangements of conditions for each S are shown in Appendix A.

Average running time for the entire experiment was 1 3/4 hours per S.

RESULTS

Scale values were taken directly from Ss' responses. Geometric means were used when pooling more than one response. Scale values were thus available from each trial (one response to each stimulus) and combinations of trials for each S in each condition.

Logarithms were taken of the scale values and were processed with the corresponding log stimulus values on an IBM 1620 computer to yield the slope, intercept, and sum of squared residuals for a least squares best fit line of the form $\log \psi = n \log \phi + \log k$, where ψ is the scale value and ϕ is the stimulus value. The slope, n, is the exponent of the psychophysical power function. Eleven exponents (n) were obtained for each S on each condition: (a) an individual exponent based upon all eight trials combined; (b) an order-1 exponent based upon the first four trials combined; (c) an order-2 exponent based upon the last four trials combined; and (d) eight exponents, one from each of the eight trials.

Responses pooled over Ss produced eleven further sets of group scale values and group exponents for each condition.

The usual analysis of magnitude production data is "backwards." In magnitude production the psychological value is given by the E and the response is the physical value (force in the present instance). Thus in terms of the present experimental procedure the psychological variable was independent and the physical variable was dependent. However, in the construction of a psychophysical function from this data the variables were

given reversed roles. A numerical magnitude was expressed as a function of a force of handgrip. Thus the usual role of independent and dependent variables was reversed.

Individual Results

Individual exponents.--Table 1 presents individual exponents (all trials pooled) for each condition. Table 2 presents summary information about the data of Table 1. Mean and median individual exponents and the group exponents (see Table 9) were similar within any condition. Only the handgrip exponents exhibited a degree of skewness.

The semi-interquartile range for handgrip was consistent with that reported by Stevens & Mack (1959) for magnitude estimation of handgrip but almost half as large as their figure for magnitude production of handgrip (see above).

Log individual scale values and their log-log plots are available in Appendices B and C. Visual examination of the log-log plots in Appendix C showed that for most individuals the linear function was an adequate first order description of the data.

Several of the plots for CMM of circle size (Appendix C 4) were clearly deviant from linearity. The tendency towards positive acceleration was also seen in the plot of the group data for this condition (Fig. 2). There was also a suggestion of an upward curvature in the plots of individual magnitude estimations of circle size (Appendix C 1). This hint suggests

TABLE 1

INDIVIDUAL EXPONENTS

Subject	Condition				
	1 Circle Size	2 Numerosity	3 Force of handgrip	4 CMM Circle Size	5 CMM Numerosity
1	1.25	0.93	1.36	0.65	0.59
2	1.23	0.70	2.79	0.76	0.50
3	1.11	0.79	0.89	1.03	0.82
4	1.61	0.63	1.83	0.71	0.60
5	1.29	0.82	1.55	0.64	0.94
6	1.22	0.94	1.16	1.07	1.28
7	1.24	0.70	0.87	1.27	1.55
8	0.70	0.61	1.06	0.77	0.67
9	1.08	1.00	0.97	0.91	0.90
10	1.07	0.68	1.37	0.75	0.58
11	1.13	0.75	0.89	0.96	0.97
12	1.07	1.04	0.96	0.97	1.24
13	1.10	0.72	1.50	0.47	0.61
14	0.97	0.70	0.95	0.88	1.12
15	1.16	0.93	0.86	1.00	1.14
16	0.98	0.91	1.70	0.72	0.66
17	1.41	0.74	1.19	0.77	0.70
18	0.96	0.91	0.85	1.00	1.08
19	1.36	0.90	1.46	1.02	0.95
20	0.93	0.89	0.74	1.24	1.30
21	0.82	0.68	1.16	0.98	0.84
22	0.80	0.90	0.99	0.52	0.44
23	1.30	0.76	0.91	0.83	0.79
24	1.22	1.13	1.24	0.47	0.60

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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TABLE 2

CENTRAL TENDENCY AND DISPERSION OF

INDIVIDUAL EXPONENTS

Measure	Condition				
	1	2	3	4	5
Range	1.61-0.70	1.13-0.61	2.79-0.74	1.24-0.47	1.55-0.44
Mean	1.125	0.823	1.217	0.850	0.870
Median	1.120	0.805	1.110	0.855	0.830
Q_1	0.975	0.706	0.900	0.715	0.600
Q_3	1.265	0.920	1.415	1.000	1.100
$(Q_3-Q_1)^{1/2}$	0.195	0.107	0.255	0.158	0.250
Standard Deviation	0.222	0.138	0.445	0.217	0.293

that the obtained curvilinearity was some sort of stimulus function, perhaps stimulus spacing, which was enhanced by the handgrip response media.

Table 3 presents intermodal correlations of individual exponents. The diagonal elements are correlations of individual order-1 with order-2 exponents and served as an indication of the stability of a S's responses.

Conditions involving responses via a squeeze of handgrip intercorrelated highly. Other conditions, especially those using the same stimuli, failed to achieve a significant correlation.

Similar analyses of the order-1 and the order-2 exponents revealed the same pattern of correlations.

The negative signs accompanying the high correlations between handgrip and CMM conditions were a result of the reversed way in which handgrip exponents were computed. No importance can be assigned to this reversal in direction of correlation.

Individual Cross Modality Matches and Predictions.--There were in the data several possible ways of obtaining a predicted individual CMM exponent. The analysis presented consists of correlations of several of the more interesting sets of predictors with the principal sets of obtained CMM exponents. These correlations appear in Table 4.

The principal findings of Table 4 were the correlations in Block A. These indicated that there is a moderate but significant degree of predictiveness in the CMM theory. The second group of correlations (Block B) confirmed the findings of Block A using various sets of predictions and obtained exponents.

TABLE 3

INTERCORRELATIONS OF INDIVIDUAL EXPONENTS

Condition	1	2	3	4	5
1. Mag. est. circles	.67**	.00	.36	-.06	-.01
2. Mag. est. dots		.73**	.24	-.02	.17
3. Mag. prod. handgrip			.83**	-.55**	-.44*
4. CMM circles				.98**	.84**
5. CMM numerosity					.68**

* Significant at $p < .05$

** Significant at $p < .01$

NOTE: Diagonal entries are reliabilities of exponents obtained by correlating a S's order-1 with his order-2 exponent.

TABLE 4

INTERCORRELATIONS OF PREDICTED vs. OBTAINED CROSS MODALITY EXPONENTS

	Predicted Exponents	Obtained Exponents	Correlation
Block A	2i/3i	5i	.496
	1i/3i	4i	.578
Block B	2i/3i	5iT1	.618
	2i/3i	5iO1	.680
	2i/3i	5iO2	.507
	2iO1/3iO1	5i	.468
	2iO1/3iO1	5iO1	.570
	1i/3i	4iO1	.450
	1i/3i	4iO2	.526
	1i/3i	4iT1	.535
Block C	2g/3i	5i	.625
	2g/3i	5iO1	.725
	1g/3i	4i	.600
	1g/3i	4iO1	.574

NOTE:--Sets of exponents are labelled in the following manner: Conditions are numbered from 1 through 5 as in previous tables (e.g., Tables 1 and 3). Individual and group exponents are indicated by i and g respectively. Order-1 and order-2 exponents are indicated by O1 and O2. T1 refers to trial one.

Thus the entry in column one, row one of "2i/3i" is read as "the set of predicted exponents consists of the set of ratios of individual exponents for numerosity to the individual exponents for handgrip." The entry in row one of column two, "5i", indicates that the obtained exponents consist of the set of individual exponents for cross modality matching of handgrip to numerosity.

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[illegible]	[illegible]	[illegible]	[illegible]
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In Block C of Table 4, the predicted exponents were obtained by dividing the group circle and numerosity exponents by the individual handgrip exponents. The correlations were, in essence, the correlation of the reciprocals of the individual handgrip exponents (times a constant) with the individual CMM exponents.

Since, in the case where two variables (e.g., x and y) are perfectly correlated, the linear regression coefficient of y on x is equal to the reciprocal of the linear regression coefficient of x on y, taking the reciprocal of the handgrip exponent is an estimate of the handgrip exponent that would be obtained were the dependent and independent variables of magnitude production data treated in the usual manner as discussed previously. The correlations of Block C, Table 4, were of interest because they were slightly higher than the correlations of handgrip and CMM conditions of Table 3.

Individual Order Exponents.--Individual order-1 and order-2 exponents for each condition are presented in Appendix D. Analyses of variance were performed on these exponents in the manner of Jones & Marcus (1961). A mixed effects model was used for these analyses. The expected values and appropriate error terms are indicated in Appendix E. This was the same model used by Jones & Marcus.

The results for the single modality analysis (circle size, numerosity, and force of handgrip) are shown in Table 5. These were similar to Jones & Marcus' results. (See Table 2, Jones & Marcus, 1961.) The Subject, Modality, and Subject X Modality interaction effects were significant.

TABLE 5

ANALYSIS OF VARIANCE OF SINGLE MODALITY
INDIVIDUAL ORDER EXPONENTS

Source	df	MS	F
Order (O)	1	.00120	-
Modality (M)	2	2.05410	13.2258*
Subjects (S)	23	.19846	8.5654*
O X M	2	.00620	-
O X S	23	.02317	1.2176
M X S	46	.16180	8.5024*
O X M X S	46	.01903	

(*$p < .01$)

Following the discussion and procedures outlined by Jones & Marcus (see above), the skewed handgrip data were eliminated and further analyses of the circle and numerosity data performed. First, an analysis of variance for these two data sets was carried out to see if eliminating the handgrip exponents would alter the results. No change in the pattern of significant effects occurred. Next each circle size exponent was multiplied by a constant equal to the ratio of the means of the two conditions. Analysis of the adjusted circle size with the non-adjusted numerosity exponents is shown in Table 6.

As was expected, the adjustment process eliminated the modality differences. The subject differences remained. In contrast to the Jones & Marcus results the Subject X Modality interaction remained highly significant. If the multiplicative interpretation was applicable to this data, the interaction would have disappeared.

An analysis of the CMM exponents is reported in Table 7.

The apparent lack of significant differences between the two modalities in Table 7 was a clear result of the Subject X Modality interaction. This was also seen in Table 1 by taking the differences between row entries in columns four and five. The mean of these differences was 0.03, which was the difference in group exponents for these conditions. However, the mean change (absolute difference) was 0.13. The interaction indicated that some Ss used a broader range of handgrip force to describe circles than they did to describe numerosity, and that for other Ss the reverse was true. The multiplicative interpretation was not relevant to this interaction.

TABLE 6

ANALYSIS OF VARIANCE OF ADJUSTED SINGLE MODALITY INDIVIDUAL ORDER EXPONENTS

Source	df	MS	F
Order (O)	1	.00282	-
Modality (M)	1	.00015	-
Subjects (S)	23	.04121	4.3803*
O X M	1	.00960	1.4337
O X S	23	.00941	1.4050
M X S	23	.04545	6.7876*
O X M X S	23	.006696	

(* $p < .01$)

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[illegible]	[illegible]	[illegible]	[illegible]
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[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]

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TABLE 7

ANALYSIS OF VARIANCE OF CROSS MODALITY INDIVIDUAL ORDER EXPONENTS

Source	df	MS	F
Order (O)	1	.054150	5.19142**
Modality (M)	1	.006020	-
Subjects (S)	23	.228610	21.9290*
O X M	1	.000105	-
O X S	23	.010425	-
M X S	23	.025890	2.1195**
O X M X S	23	.012215	

(* $p < .01$; ** $p < .05$)

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The Order effect of Table 7 indicated that there was a slight, but significant, increase in exponent as time and number of CMM judgments increased. This result was not found in the analysis of the single modality exponents. Examination of the group exponents for each trial (shown in Table 8) confirmed this result and suggested that the Order effect is associated with the handgrip response procedure. Rows three, four, and five (the handgrip response conditions) of Table 8 appeared to vary systematically with trials.

The trial effect is illustrated in Fig. 1, where two sets of averages taken from Table 8 are plotted. These are the averages of entries in rows one and two of Table 8 (the numerical response conditions) and the averages of entries in rows three, four, and five (conditions involving a handgrip response). The former plot is essentially horizontal after the first trial. The plot for handgrip responses rises gradually.

Group Results

The group exponents are shown in Table 9. Group scale values are plotted in Fig. 2 and are available in Appendix F.

For the CMM conditions it was predicted that the obtained exponent would approximate the ratio of the exponents for the two modalities which were matched. For these data the predicted exponents, based upon the results reported in Table 9, were 0.99 for circle size and 0.72 for numerosity. The obtained exponents were 0.84 and 0.87 respectively. This error in prediction (0.15 for both modalities) was larger than any previously reported. Note that the errors for the two modalities were in opposite directions.

TABLE 8

GROUP EXPONENTS FOR EACH TRIAL AND CONDITION

Condition	Trial 1	2	3	4	5	6	7	8
1. Mag. est. circle size	1.04	1.15	1.17	1.14	1.12	1.11	1.13	1.13
2. Mag. est. numerosity	0.76	0.82	0.85	0.80	0.86	0.85	0.82	0.83
3. Mag.prod. force handgrip	1.03	1.11	1.14	1.18	1.08	1.14	1.12	1.20
4. CMM circle size	0.79	0.80	0.86	0.79	0.85	0.85	0.92	0.86
5. CMM numerosity	0.82	0.78	0.87	0.84	0.98	0.90	0.89	0.90

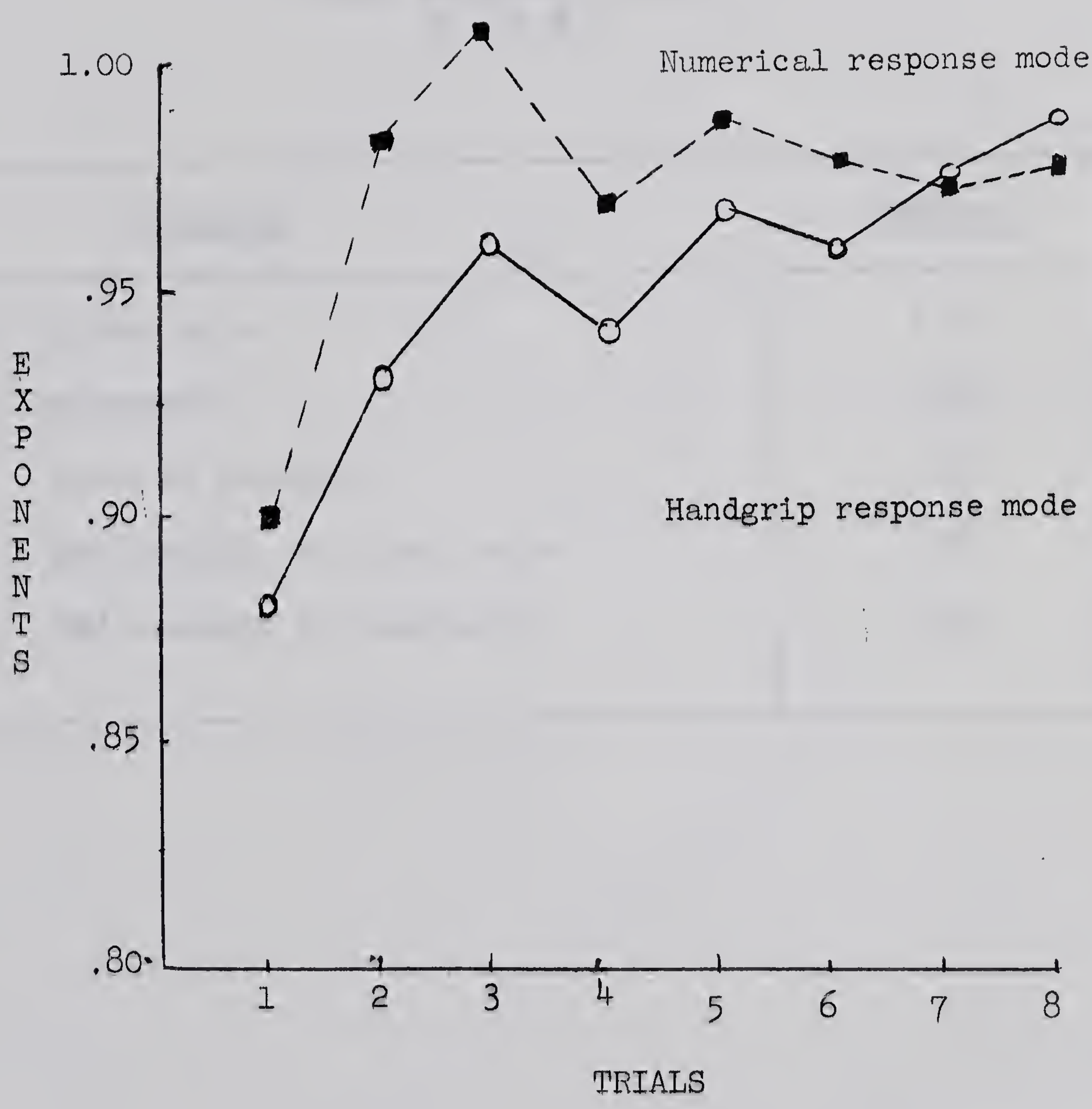


Fig. 1. Average group exponents for each trial. Numerical and handgrip response modes.

TABLE 9

GROUP EXPONENTS (n) IN
$\Psi = k\,\phi^{n}$

Condition	Exponent
1. Circle size	1.12
2. Numerosity	0.82
3. Force of handgrip	1.13
4. CMM handgrip to circle size	0.84
5. CMM handgrip to numerosity	0.87

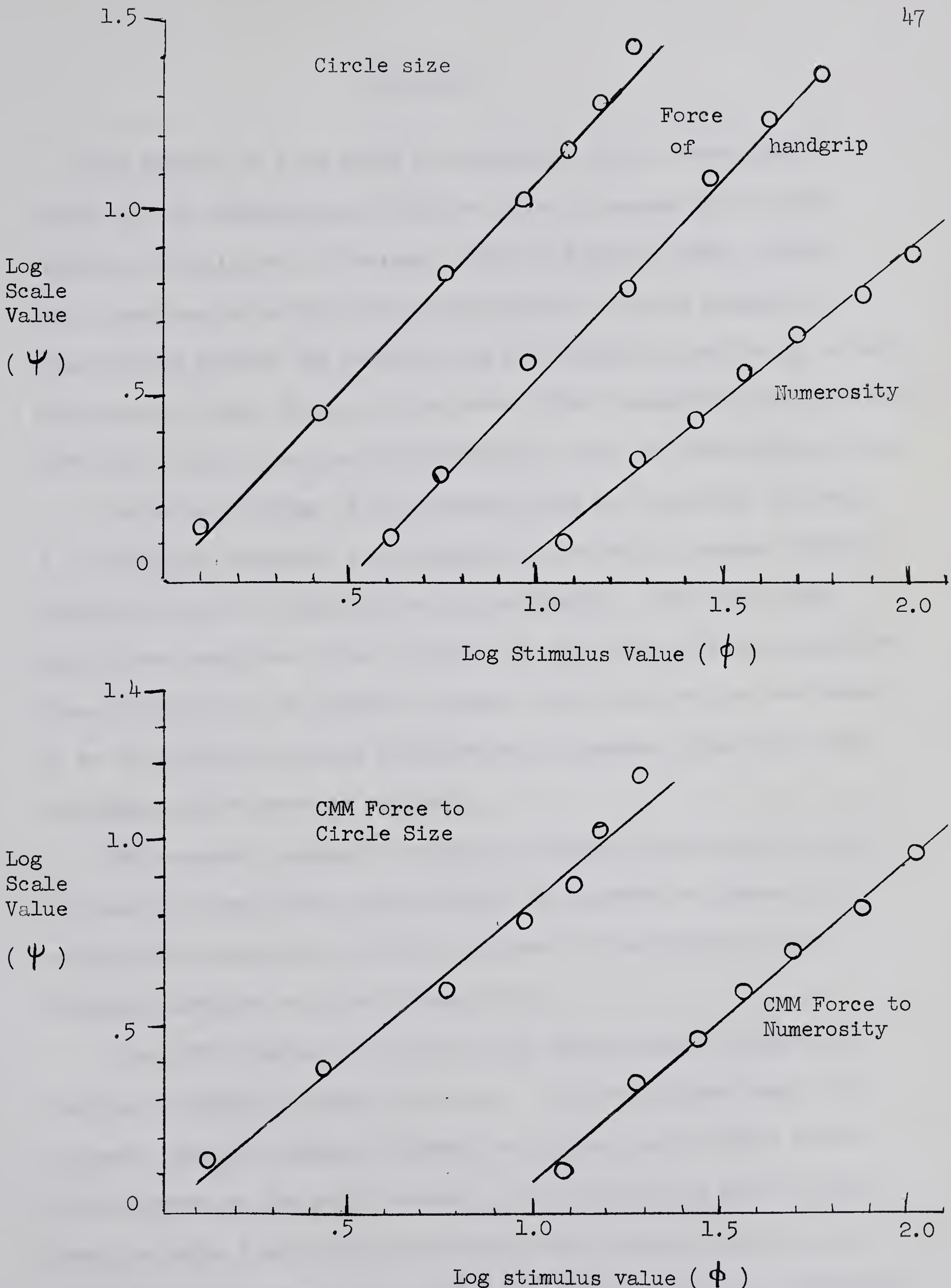
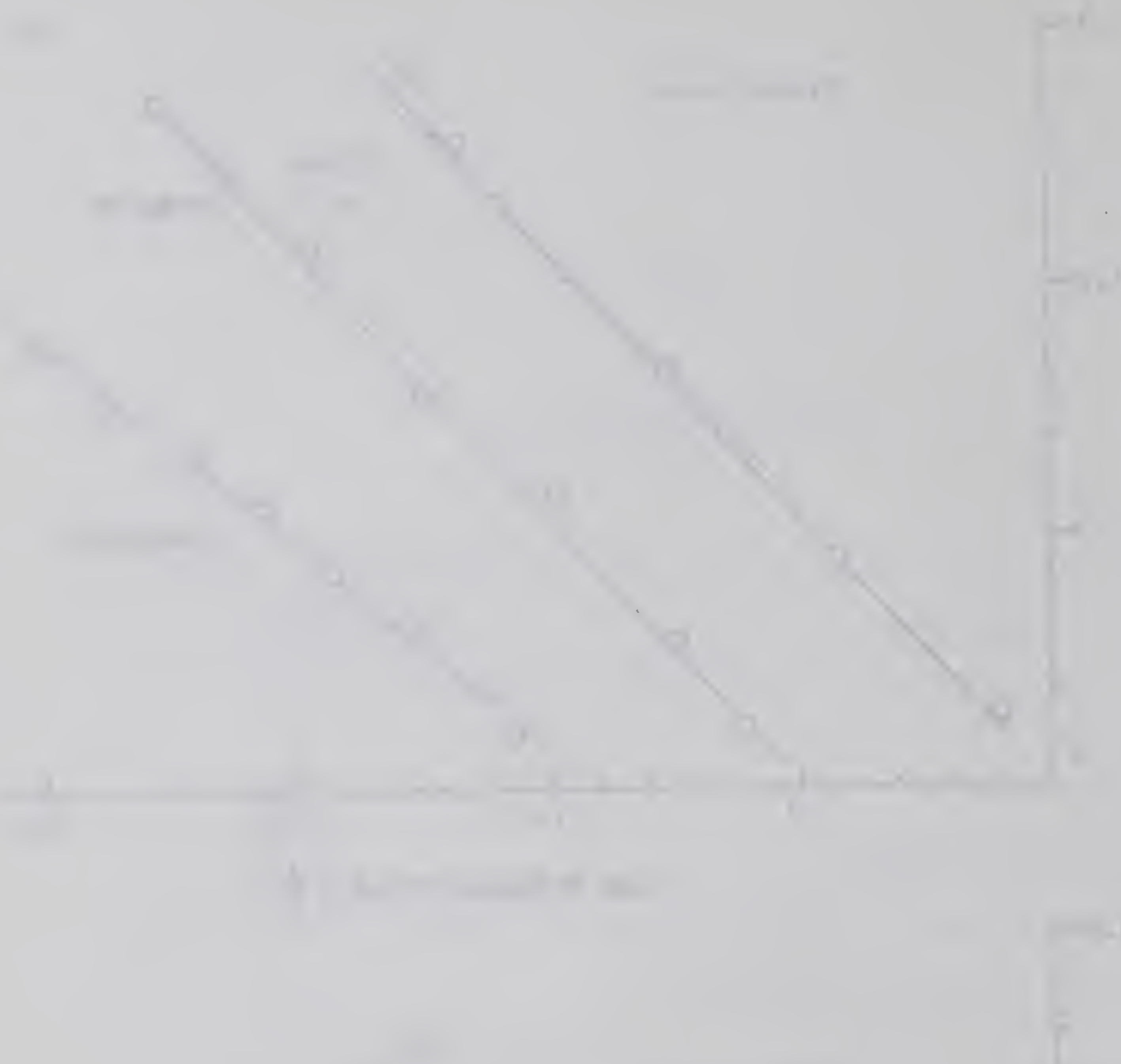


Fig. 2. Group psychophysical functions. Logarithmic co-ordinates. Relative positions along both axes are arbitrary.



DISCUSSION

The results of this study bear directly upon a conceptualization of the psychophysical function first discussed by Attneave (1962) and explicated by Treisman (1964) and Ekman (1964a, 1964b). Their position holds that the psychophysical function expresses a relationship between the stimulus and the response class the S is using. Furthermore, Ekman, Hosman, & Lindström (1964) suggested that individual differences are a function of differences in use of the response class.

The major findings of the present study are contained in Table 3. Individual exponents from conditions involving a common response language (force of handgrip) correlated highly. Individual exponents from conditions using a common set of stimuli did not correlate. These correlations of handgrip response conditions may be considered to be an indication of the contribution of response language to the individual differences in exponents.

The response language view may be further demonstrated by the increase in correlation--and therefore an increase in linearity of relationship--observed in Table 4 between the reciprocal of the handgrip exponents and the CMM exponents.

Group CMM studies (see above) have demonstrated the effect of response language on group exponents. The results have shown that a change from one response language to another had a simple quantitative effect on the group exponent. The significant Order effect found in Table 7 and the insignificant Order effects reported by Jones & Marcus suggest that there were as well qualitative differences

in response languages. The Order variable was not a factor when a magnitude estimation response language was used but the Order variable was of significance when force of handgrip was used as a response language. The exact nature of this Order effect is unclear since the order of stimuli within a trial was confounded with the number of trials in both the present study and that of Jones & Marcus.

The response language argument would seem to be threatened by the zero correlation between the magnitude estimation of circle size and of numerosity conditions. However, reference to the instructions given for these conditions reveals that, although the Ss were required to emit numerical responses in both conditions, the responding procedures differed.

In making judgments of circle size the S followed the usual magnitude estimation no-prescribed-modulus procedure. This response mode may be referred to as a magnitude estimation response language. When estimating numerosity, Ss were instructed to respond in terms of the actual number of dots present. In making a judgment of this sort the S may be using a response language different from the magnitude estimation response language in that the S attempts to respond in terms of a physical unit of the stimulus. Such a mode may be referred to as a natural response language.

The literature does not suggest that this distinction between types of numerical response languages should be made. The principal parametric studies of magnitude estimation have been concerned with

subjective loudness (S. S. Stevens, 1956), a modality for which no clear-cut physical unit is available to the subject. A natural response language procedure used to study apparent length and proportionality (S. S. Stevens & Galanter, 1957) was referred to as magnitude estimation (see also Galanter, 1963).

No one has previously compared a natural response and a magnitude estimation response language procedure on the same continua. Concurrently with the present study, Morris & Rule (1965) conducted a study of instructional set and response languages. Under numerosity instructions they obtained a group magnitude estimation no-prescribed-modulus exponent of 0.98 and a group natural response exponent of 0.83.

The analyses of Tables 5 and 6 failed to repeat the multiplicative results of Jones & Marcus. This failure could be attributed to the confounding of response languages in the circle size and numerosity conditions. Thus the result may be considered additional support for the indentification of separate response languages operating in these conditions.

In the analysis of the CMM conditions (Table 7) the response language variable was not confounded. However, the insignificant modality effect did not permit a test of the Jones & Marcus multiplicative theory.

The exponent of the group numerosity function was 0.82. This figure differs from all other as yet published data for this continua. S. S. Stevens & Galanter (1957) report on exponent of 1.34 obtained

via the method of fractionation. Abbey (1962) obtained a magnitude estimation no-prescribed-modulus exponent of 1.20 from five Ss. Strangert (1961) reported nine ratio estimation exponents ranging from 0.99 to 1.97 as a function of stimulus dispersion. The exponent (0.82) reported in the present study falls well below his curve.

That the group exponent (0.82) was clearly less than 1.00 is consistent with classical psychophysical studies of absolute numerosity (Burnett, 1906) in that there was an increasing error of underestimation as the size of the stimulus increased.

The inconsistency of this data with previous scaling reports can be attributed to the differences in response languages. This inconsistency also indicates that there is a lack of invariance across scaling techniques for the continuum of numerosity.

The group exponents for circle size and force of handgrip (Table 9) supported previous evidence reported by Strangert (1961) regarding the value of the exponent and the dispersion of the stimuli used to obtain the exponent. Strangert plotted thirty exponents from six continua against the standard deviation of the logs of the stimulus values. The exponents varied inversely and non-linearly with dispersion of the stimuli.

The exponent for subjective size of circles, 1.12, was in satisfactory agreement with previous published studies of this modality. S. S. Stevens (1957) reported that the exponent for visual area ranges from 0.90 to 1.15. Strangert (1961) reported eight exponents for

circular areas ranging from 0.77 to 1.20. The exponent obtained in the present study when plotted with the standard deviation of the stimulus series fell directly onto Strangert's plot.

S. S. Stevens & Guirao (1963) obtained an exponent for largeness of square areas of 0.70. The instructions for this particular portion of their study were not reported. If their Ss were asked to judge largeness then their results indicated that largeness instructions may produce a different function than size instructions.

The exponent, 1.13, obtained for force of handgrip differs markedly from previous results (see above). It is suggested that the differentiating factor was the range of numbers to which Ss responded by handgrip pressure. The stimuli used by Stevens & Mack (1959) were 3, 6, 10, 20, and 30. The stimulus series in the present study was extended in both directions. The exponents obtained in the present study and by Stevens & Mack fell directly onto the trend of Strangert's plot of stimulus dispersion and the value of the exponent.

Only moderate correlations between predicted and obtained CMM exponents and unusually large errors in group predictions were found in the present study. These results suggest that there may be some significance in Poulton & Simmonds' (1963) evaluation of CMM that

> ...it only demonstrates that experienced O's (Stevens, Mack, & Stevens, 1960, p. 61) can learn to judge consistently in and between different sensory continua.
>
> (Poulton & Simmonds, 1963, p. 303)

Ss in the present study were relatively inexperienced in making psychophysical judgments. The problem, unsolvable without further systematic research, is thus that of population differences, if any, and of the effects of varied amounts of experience in making psychophysical judgments (see also Sternbach & Tursky, 1964).

Table 8 and Fig. 1 indicate that there were trial-to-trial effects. Despite the confounding of trials and order of stimuli within trials, there is evidence for some sort of a warm-up effect. The change was most dramatic in the numerical response conditions. What these trial effects mean and whether they were a shift toward or away from a veridical function for sensation is a question to be explored. Information on this point would have a bearing on interpretation of Poulton & Simmonds' (1963) widely varying one-trial results.

SUMMARY

Individual psychophysical functions were obtained from 24 Ss for each of five tasks: (a) magnitude estimation of circle size, (b) magnitude estimation of numerosity, (c) magnitude production of force of handgrip, (d) cross modality matching of handgrip to circle size, and (e) cross modality matching of handgrip to numerosity.

Individual differences were found to be highly reliable.

Correlations over Ss indicated that response language was a significant contributor toward individual variability in the exponent of the psychophysical function.

Evidence was obtained indicating that there is a natural response language distinct from a magnitude estimation response language although both require Ss to emit numbers.

Cross modality predictions were only moderately successful. The CMM exponents exhibited a significant Subject X Modality interaction that did not support a multiplicative components hypothesis.

In four of the five conditions there was no systematic deviation from linearity in individual log-log plots of the psychophysical function. This result was not considered to be a challenge to the use of the power function as a first order description of psychophysical judgment data.

References

Abbey, D. Cross modality matching of numerosity and pitch. Canad. J. Psychol., 1962, 16, 283-289.

Attneave, F. Perception and related areas. In S. Koch (Ed.) Psychology: A study of a science. Vol. 4. New York: McGraw-Hill, 1962, 619-659.

Beck, J. & Shaw, W. A. The scaling of pitch by the method of magnitude estimation. Amer. J. Psychol., 1961, 74, 242-251.

Björkman, M. & Strangert, B. The relationship between ratio estimates and stimulus dispersion. Rep. Psychol. Lab., Univer. Stockholm, 1960, No. 81.

Boring, E. G. A history of experimental psychology. New York: Appleton-Century-Crofts, 1957.

Brown, J. S. The motivation of behavior. New York: McGraw-Hill, 1961.

Burnett, C. T. The estimation of number. Harvard Psychological Studies, 1906, 2, 349-404.

Corso, J. A theoretico-historical review of the threshold concept. Psychol. Bull., 1963, 60, 356-370.

Edwards, A. Techniques of attitude scale construction. New York: Appleton-Century-Crofts, 1957.

Eisler, H. Similarity in the continuum of heaviness with some methodological and theoretical considerations. Scand. J. Psychol., 1960, 1, 69-81.

Eisler, H. On the problem of category scales in psychophysics. Scand. J. Psychol., 1962, 3, 81-87. (a)

Eisler, H. Empirical test of a model relating magnitude and category scales. Scand. J. Psychol., 1962, 3, 88-96. (b)

Eisler, H. Subjective scale of force for a large muscle group. J. exp. Psychol., 1962, 64, 253-257. (c)

Eisler, H. Indirect scaling. Rep. Psychol. Lab., Univer. Stockholm, 1963, No. 158. (a)

Eisler, H. Magnitude scales, category scales, and Fechnerian integration. Psychol. Rev., 1963, 70, 243-253. (b)

Eisler, H. Possible implications of a non-constant subjective zero in the psychophysical power function. Percept. mot. Skills, 1963, 16, 778. (c)

Eisler, H. A general differential equation in psychophysics: derivation and empirical test. Scand. J. Psychol., 1963, 4, 265-272. (d)

Eisler, H. How prothetic is the continuum of smell? Scand. J. Psychol., 1963, 4, 29-32. (e)

Ekman, G. Two generalized ratio scaling methods. J. Psychol., 1958, 45, 287-295.

Ekman, G. Weber's law and related functions. J. Psychol., 1959, 47, 343-352.

Ekman, G. Some aspects of psychophysical research. In W. Rosenblith (Ed.) Sensory communication. Cambridge: M.I.T. Press, 1961, pp. 35-48.

Ekman, G. The measurement of moral judgment: a comparison of scaling methods. Percept. mot. Skills, 1962, 15, 3-9.

Ekman, G. A direct method for multidimensional ratio scaling. Psychometrika, 1963, 28, 33-41. (a)

Ekman, G. Contributions to the psychophysics of color vision. Studium Generale, Jahrg. 16, Heft 1, 1963, S. 54-64. (b)

Ekman, G. Is the power law a special case of Fechner's law. Percept. mot. Skills, 1964, 19, 730. (a)

Ekman, G. A note on alternative psychophysical laws. Rep. Psychol. Lab., Univer. Stockholm, 1964, No. 175. (b)

Ekman, G., Frankenhäuser, M., Levander, S., & Mellis, I. Scales of unpleasantness of electrical stimulation. Rep. Psychol. Lab., Univer. Stockholm, 1964, No. 170.

Ekman, G., Hosman, J., & Lindström, B. Roughness, smoothness, and preference: a study of quantitative relations in individual subjects. Rep. Psychol. Lab., Univer. Stockholm, 1964, No. 173.

Ekman, G. & Künnapas, T. Scales of aesthetic value. Percept. mot. Skills, 1962, 14, 19-26. (a)

Ekman, G. & Künnapas, T. Measurement of aesthetic value by "direct" and "indirect" methods. Scand. J. Psychol., 1962, 3, 33-39. (b)

Ekman, G. & Künnapas, T. A further study of direct and indirect scaling methods. Scand. J. Psychol., 1963, 4, 77-80. (a)

Ekman, G. & Künnapas, T. Scales of masculinity and femininity: a further study of direct and indirect scaling methods. Rep. Psychol. Lab., Univer. Stockholm, 1963, No. 162. (b)

Ekman, G. & Künnapas, T. Scales of conservatism. Percept. mot. Skills, 1963, 16, 329-334. (c)

Engen, T. & Lindström, C. O. Psychophysical scales of the odor intensity of amyl acetate. Scand. J. Psychol., 1963, 4, 23-28.

Galanter, E. Contemporary psychophysics. In R. Brown, E. Galanter, E. Hess, & G. Mandler, New directions in psychology. New York: Holt, Rinehart, & Winston, 1962.

Garner, W. R. Context effects and the validity of loudness scales. J. exp. Psychol., 1954, 48, 218-224.

Garner, W. R. The development of context effects in half loudness judgments. J. exp. Psychol., 1959, 58, 212-219.

Garner, W., Hake, H., & Eriksen, C. Operationism and the concept of perception. Psychol. Rev., 1956, 63, 149-159.

Helson, H. Adaptation-level theory: an experimental and systematic approach to behavior. New York: Harper & Row, 1964.

Jones, F. N. & Marcus, M. Subject effects in judgments of subjective magnitude. J. exp. Psychol., 1961, 61, 40-44.

Jones, F. N. & Woskow, M. J. Some effects of context on the slope in magnitude estimation. J. exp. Psychol., 1966 (in press).

Künnapas, T. Scales for subjective distance. Scand. J. Psychol., 1960, 1, 187-192.

Künnapas, T. & Sillen, M. The measurement of political preferences: a comparison of scaling methods. Rep. Psychol. Lab., Univer. Stockholm, 1964, No. 172.

Lewis, D. Quantitative methods in psychology. New York: McGraw-Hill, 1960.

Luce, R. D. & Galanter, E. Psychophysical scaling. In R. Luce, R. Bush, & E. Galanter, (Eds.) Handbook of mathematical psychology, Vol. I, New York: Wiley, 1963, pp. 245-308.

Michels, W. & Helson, H. A reconciliation of the veg scale with Fechner's law. Amer. J. Psychol., 1954, 67, 677-683.

Morris, R. & Rule, S. J. The effects of response anchors on psychophysical functions. Paper read at Canadian Psychological Association, Vancouver, June, 1965.

Poulton, E. C., & Simmonds, D. C. V. Value of standard and very first variable in judgments of reflectance of grays with various ranges of available numbers. J. exp. Psychol., 1963, 65, 297-304.

Pradhan, P. L. & Hoffman, P. Effects of spacing and range of stimuli on magnitude estimation judgments. J. exp. Psychol., 1963, 66, 533-541.

Scheffé, H. A method for judging all contrasts in the analysis of variance. Biometrika, 1953, 40, 381-400.

Sjöberg, L. The law of categorical judgment without the assumption of equal variances and covariances. Rep. Psychol. Lab., Univer. Stockholm, 1962, No. 140.

Sjöberg, L. An empirical application of a new case of the law of comparative judgment. Scand. J. Psychol., 1963, 4, 97-107.

Sternbach, R. & Tursky, B. On the psychophysical power function in electric shock. Psychonomic Science, 1964, 1, 217-218.

Stevens, J. C. Stimulus spacing and the judgment of loudness. J. exp. Psychol., 1958, 56, 246-250.

Stevens, J. C. & Mack, J. D. Scales of apparent force. J. exp. Psychol., 1959, 58, 405-413.

Stevens, J. C., Mack, J. D., & Stevens, S. S. Growth of sensation on seven continua as measured by force of handgrip. J. exp. Psychol., 1960, 59, 60-67.

Stevens, J. C. & Stevens, S. S. Warmth and cold: the dynamics of sensory intensity. J. exp. Psychol., 1960, 60, 183-192.

Stevens, S. S. The measurement of loudness. J. acoust. Soc. Amer., 1955, 27, 815-829.

Stevens, S. S. The direct estimation of sensory magnitudes: loudness. Amer. J. Psychol., 1956, 69, 1-25.

Stevens, S. S. On the psychophysical law. Psychol. Rev., 1957, 64, 153-181.

Stevens, S. S. Cross-modality validation of subjective scales of loudness, vibration, and electric shock. J. exp. Psychol., 1959, 57, 201-209. (a)

Stevens, S. S. Tactile vibration: the dynamics of sensory intensity. J. exp. Psychol., 1959, 57, 210-219. (b)

Stevens, S. S. Toward a resolution of the Fechner-Thurstone legacy. Psychometrika, 1961, 26, 35-47. (a)

Stevens, S. S. The psychophysics of sensory function. In W. Rosenblith (Ed.), Sensory communication. Cambridge: M.I.T. Press, 1961, pp. 1-34. (b)

Stevens, S. S. The surprising simplicity of sensory metrics. Amer. Psychologist, 1962, 17, 29-39.

Stevens, S. S., Carton, A. S., & Shickman, G. M. A scale of apparent intensity of electric shock. J. exp. Psychol., 1958, 56, 328-333.

Stevens, S. S. & Galanter, E. Ratio scales and category scales for a dozen perceptual continua. J. exp. Psychol., 1957, 54, 377-411.

Stevens, S. S. & Guirao, M. Subjective scaling of length and area an the matching of length to loudness and brightness. J. exp. Psychol., 1963, 66, 177-187.

Stevens, S. S. & Harris, J. R. The scaling of subjective roughness and smoothness. J. exp. Psychol., 1962, 64, 489-494.

Strangert, B. A validation study of the method of ratio estimation. Rep. Psychol. Lab., Univer. Stockholm, 1961, No. 95.

Thurstone, L. L. A law of comparative judgment. Psychol. Rev., 1927, 34, 273-286.

Titchener, E. B. Experimental psychology, quantitative. New York: Macmillan, 1905, pp. 20-116.

Torgerson, W. Theory and methods of scaling. New York: Wiley, 1958.

Treisman, M. Sensory scaling and the psychophysical law. Quart. J. exp. Psychol., 1964, 16, 11-22.

White, E. Subjectively equated stimulus intensities and autonomic reactivity. J. exp. Psychol., 1964, 68, 297-300.

Williams, E. G. The interpretation of interactions in factorial experiments. Biometrika, 1952, 39, 65-81.

Wong, R. The effect of the modulus on estimates of the magnitude of linear extent. Amer. J. Psychol., 1963, 76, 511-512.

APPENDICES

APPENDIX A1

Stimulus Ordering (All Conditions)

Order Number (Trial)	Stimulus Number[a]						
1	4	5	1	6	3	7	2
2	4	3	6	1	2	5	7
3	4	2	5	3	1	7	6
4	5	1	2	7	3	6	4
5	4	6	3	7	2	1	5
6	6	7	1	3	5	2	4
7	7	5	2	1	6	3	4
8	2	7	3	6	1	5	4

a Stimuli are ranked by size from smallest to largest.

[illegible]

[illegible]

[illegible]							[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

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APPENDIX A2

Assignment of Conditions to Ss

Subject Number	Single Modality[1] Order	Cross Modality[1] Order
1	ABCCBA	DEED
2	BACCAB	EDED
3	ACBBCA	EDDE
4	CABBAC	DEED
5	ACBBCA	DEED
6	BCAACB	EDED
7	CABBAC	DEDE
8	CBAABC	DEDE
9	CBAABC	EDED
10	ABCCBA	DEED
11	BCAACB	EDED
12	BACCAB	EDED
13	BCAACB	DEED
14	BACCAB	DEDE
15	BCAACB	EDDE
16	CABBAC	DEDE
17	ACBBCA	EDDE
18	ACBBCA	EDED
19	ABCCBA	EDDE
20	BACCAB	DEDE
21	CABBAC	DEDE
22	CBAABC	EDDE
23	ABCCBA	DEED
24	CBAABC	EDDE

[1]A refers to Mag. Est. of Circle size

B refers to Mag. Est. of numerosity

C refers to Mag. Prod. of Force of Handgrip

D refers to CMM of Circle size

E refers to CMM of Numerosity

APPENDIX B

Individual Log Scale Values

(Mean log of eight responses)

Circle Size

Stimuli	Subject 1	2	3	4	5	6
1	0.0000	0.2049	0.0806	-0.4008	0.0376	0.1505
2	0.3230	0.4549	0.3616	0.1825	0.3891	0.5000
3	0.8185	0.7366	0.8668	0.8720	0.8010	0.8218
4	0.9601	1.1037	1.0000	1.0282	1.0000	1.1626
5	1.1498	1.3039	1.1154	1.1585	1.2919	1.3167
6	1.2649	1.4707	1.2057	1.3955	1.3635	1.4515
7	1.5416	1.6213	1.4262	1.6244	1.5794	1.5625

Stimuli	7	8	9	10	11	12
1	0.1881	0.2634	0.3010	0.1501	0.0376	0.0000
2	0.3827	0.4771	0.5534	0.3890	0.5482	0.3010
3	0.7998	0.6506	0.8451	0.6990	0.9203	0.6263
4	1.1470	0.7835	1.0000	1.0000	1.0515	0.7747
5	1.1990	0.8877	1.2297	1.1336	1.1826	0.9978
6	1.4515	0.9900	1.4573	1.2370	1.3195	1.1321
7	1.6193	1.1390	1.6085	1.3902	1.4246	1.3010

Stimuli	13	14	15	16	17	18
1	0.0000	0.4758	0.1626	0.3670	-0.0242	0.1413
2	0.3890	0.7755	0.4879	0.5326	0.2825	0.5260
3	0.7736	1.1697	0.8132	0.9503	0.7650	0.8635
4	0.9994	1.3131	1.0950	1.0880	1.0000	0.9936
5	1.0956	1.4276	1.1883	1.2606	1.2386	1.0921
6	1.1829	1.5084	1.3792	1.3756	1.4254	1.1826
7	1.2922	1.6191	1.5759	1.4848	1.6532	1.3115

Stimuli	19	20	21	22	23	24
1	-0.1371	0.1917	0.1505	0.1129	-0.0075	0.6990
2	0.3728	0.3957	0.3890	0.4331	0.3137	1.0220
3	0.6713	0.8851	0.5979	0.6726	0.7369	1.4740
4	1.0220	1.0052	0.7056	0.8840	0.9752	1.6336
5	1.2073	1.0977	0.8959	0.9493	1.1690	1.7529
6	1.3314	1.1714	1.0080	0.9857	1.3754	1.8864
7	1.5068	1.2102	1.1690	1.0473	1.5084	2.0288

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Numerosity						
Stimuli	Subject					
	1	2	3	4	5	6
1	0.9478	0.9516	0.9656	0.9753	0.9979	0.9886
2	1.1673	1.2030	1.2112	1.1442	1.1948	1.1618
3	1.3080	1.4012	1.3683	1.1919	1.3406	1.3236
4	1.4542	1.3826	1.4930	1.3008	1.4630	1.4034
5	1.5639	1.5927	1.5713	1.3936	1.5418	1.6501
6	1.7182	1.5645	1.6801	1.4960	1.7042	1.7144
7	1.8490	1.6698	1.7340	1.5972	1.7856	1.8841
	7	8	9	10	11	12
1	1.0090	0.9766	1.0192	0.9764	0.9472	0.9819
2	1.3230	1.1520	1.2505	1.1477	1.1754	1.1917
3	1.3670	1.2255	1.4360	1.2386	1.3507	1.4593
4	1.4324	1.3058	1.5821	1.4034	1.4386	1.5951
5	1.5638	1.4173	1.8077	1.4608	1.5386	1.6890
6	1.6604	1.4684	1.8310	1.5147	1.6060	1.9087
7	1.7386	1.5977	1.9813	1.6327	1.6844	1.9500
	13	14	15	16	17	18
1	0.9414	0.9144	0.9643	0.9599	0.9828	0.9756
2	1.1178	1.1420	1.1576	1.1560	1.2262	1.1823
3	1.2386	1.2484	1.4082	1.3435	1.3138	1.3751
4	1.3243	1.3195	1.5106	1.5422	1.4338	1.5261
5	1.4018	1.4375	1.6924	1.5979	1.5154	1.5980
6	1.5619	1.5141	1.7568	1.6851	1.6162	1.7694
7	1.6375	1.6263	1.8295	1.8468	1.7333	1.8428
	19	20	21	22	23	24
1	1.0000	1.0088	0.9707	1.0212	0.9828	0.9478
2	1.1697	1.2566	1.1519	1.3252	1.1541	1.2023
3	1.2628	1.3472	1.2484	1.4276	1.2854	1.2842
4	1.4254	1.4593	1.3074	1.5167	1.4388	1.4945
5	1.5187	1.6360	1.4573	1.6497	1.5240	1.7022
6	1.6644	1.7668	1.5502	1.7789	1.6107	1.8463
7	1.9034	1.8657	1.6312	1.9522	1.7089	2.0294

Force of Handgrip						
Stimuli	Subject					
	1	2	3	4	5	6
1	1.0929	1.3585	0.5328	1.0269	0.4952	0.8183
2	1.1910	1.4298	0.6578	1.2037	0.4592	0.9212
3	1.4268	1.5577	0.9908	1.2577	0.7430	1.2348
4	1.6210	1.6510	1.1583	1.4707	0.9396	1.4461
5	1.8512	1.7355	1.6154	1.5890	0.9679	1.5698
6	1.8298	1.7728	1.8013	1.6523	1.1739	1.8184
7	2.0670	1.8306	1.9369	1.7725	1.3061	1.9442
	7	8	9	10	11	12
1	0.4355	0.8091	0.3979	0.7054	0.4732	0.4732
2	0.6237	0.9615	0.5484	0.9120	0.5108	0.6451
3	0.7963	1.3530	0.8339	1.1464	0.9276	0.9312
4	1.1184	1.5325	1.0724	1.2820	1.3396	1.3526
5	1.5285	1.8243	1.4215	1.4783	1.5615	1.5129
6	1.7612	1.9178	1.5658	1.5759	1.7014	1.6699
7	1.8807	1.9970	1.7304	1.7220	1.7966	1.7727
	13	14	15	16	17	18
1	1.0271	0.4355	0.3979	1.0249	0.9908	0.3979
2	1.1325	0.4355	0.4355	1.1811	1.0526	0.4355
3	1.3513	0.6614	0.7162	1.3715	1.2309	0.6375
4	1.5248	1.0029	1.2947	1.5134	1.5128	1.0561
5	1.6546	1.3101	1.4613	1.5616	1.7674	1.4070
6	1.7944	1.4720	1.5782	1.7466	1.8926	1.5645
7	1.9069	1.7325	1.7772	1.8188	2.0354	1.8289
	19	20	21	22	23	24
1	0.4355	0.4355	0.9771	0.5705	0.5485	0.9615
2	0.5485	0.6081	1.1633	0.6081	0.8984	1.2188
3	0.6457	0.7806	1.3832	0.9866	0.9835	1.4767
4	0.9376	1.0625	1.6398	1.3169	1.2975	1.6434
5	1.1024	1.6944	1.8355	1.5528	1.6342	1.8217
6	1.1598	1.9318	2.0104	1.6576	1.8422	1.9665
7	1.3152	2.0640	2.0954	1.7908	2.0149	2.0481

CMM Circle Size						
Stimuli	Subject					
	1	2	3	4	5	6
1	1.1811	0.9376	0.4355	0.7210	0.3979	0.5484
2	1.3177	1.0782	0.6990	1.0865	0.4355	0.7742
3	1.4799	1.3291	1.0113	1.2654	0.5484	1.1085
4	1.6280	1.4865	1.1803	1.3935	0.6834	1.3543
5	1.7182	1.5969	1.3528	1.4326	0.7650	1.4854
6	1.8497	1.7054	1.5093	1.5055	1.1028	1.6115
7	1.9656	1.8463	1.6803	1.6333	1.1598	1.8342
	7	8	9	10	11	12
1	0.3979	0.9248	0.3979	0.5484	0.5704	0.5485
2	0.5861	1.2058	0.3979	0.8027	0.8026	0.8843
3	0.7870	1.4520	0.5108	0.9895	1.2327	1.1264
4	1.2676	1.5526	0.8403	1.2068	1.3859	1.3505
5	1.3693	1.6638	1.0526	1.2415	1.4631	1.4969
6	1.6822	1.7581	1.2225	1.3461	1.5791	1.5951
7	1.9093	1.8808	1.4771	1.4505	1.6718	1.7004
	13	14	15	16	17	18
1	1.1413	0.4732	0.5484	0.9369	1.0902	0.4355
2	1.3825	0.6457	0.8339	1.0989	1.4781	0.6834
3	1.4336	0.8779	1.1301	1.2960	1.6012	0.8935
4	1.6332	1.9038	1.3499	1.4912	1.8295	1.1932
5	1.6138	1.1462	1.3459	1.5186	1.9189	1.2593
6	1.6368	1.3391	1.6460	1.6716	1.9579	1.4792
7	1.7518	1.5766	1.7800	1.8063	2.0124	1.6525
	19	20	21	22	23	24
1	0.3979	0.4732	0.7586	1.2737	0.9376	1.3260
2	0.5861	0.6116	1.3586	1.5362	1.1817	1.6074
3	0.7586	0.8403	1.6044	1.6121	1.3148	1.6657
4	1.0185	1.2909	1.7765	1.6822	1.4872	1.7682
5	1.1739	1.5295	1.8154	1.7923	1.6030	1.7758
6	1.3847	1.7550	1.9347	1.8696	1.8616	1.8354
7	1.7193	1.8318	1.9968	1.9388	1.9755	1.9908

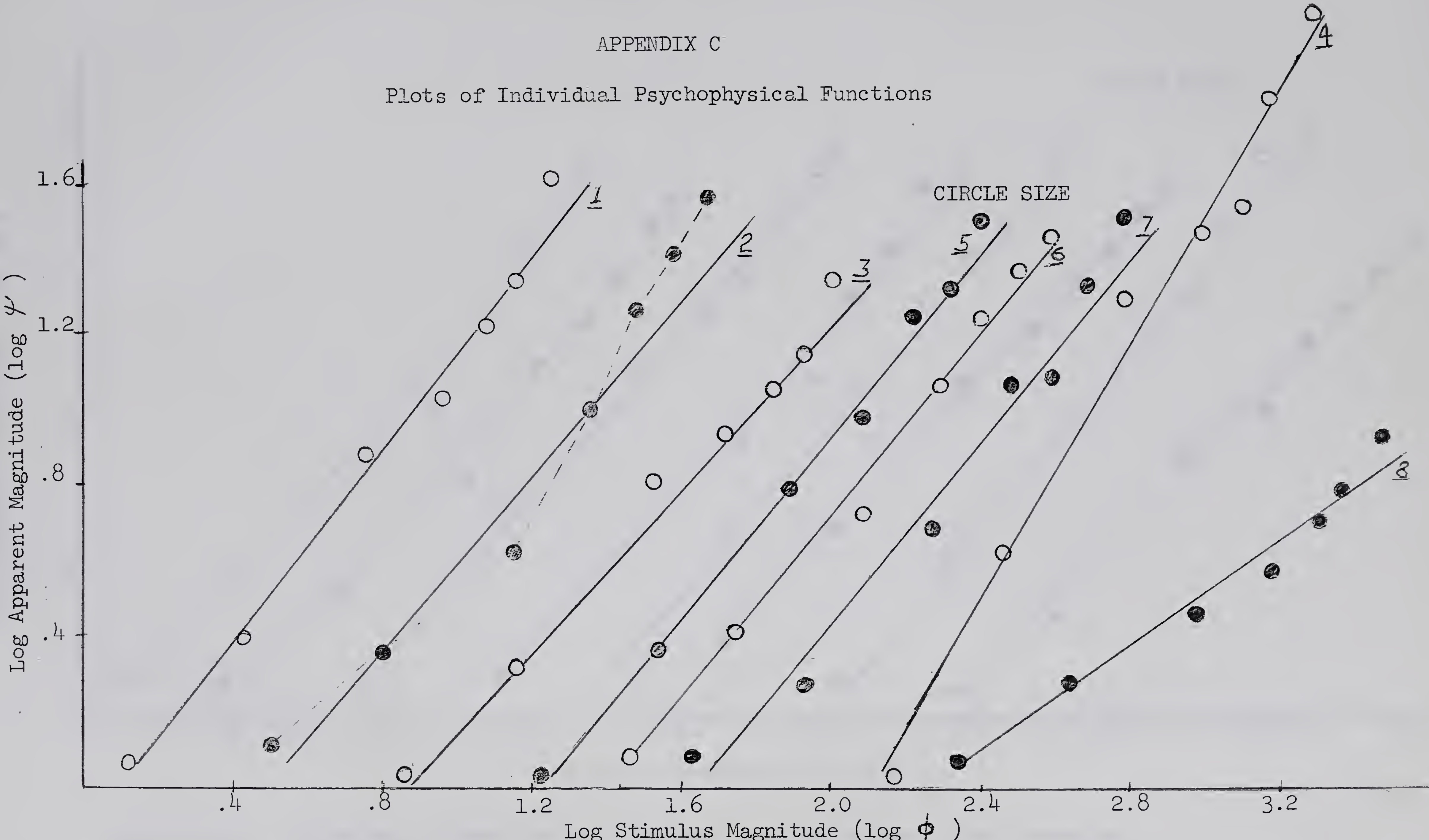
[illegible]

CMM Numerosity						
Stimuli	Subject					
	1	2	3	4	5	6
1	1.4203	1.3580	0.9276	1.0051	0.4355	0.5328
2	1.5435	1.4192	1.0903	1.2586	0.6874	0.8339
3	1.6426	1.5354	1.2370	1.3522	0.7430	1.0652
4	1.6855	1.5666	1.3316	1.4239	0.9554	1.2095
5	1.7917	1.6183	1.4197	1.4693	1.0454	1.3627
6	1.8917	1.7625	1.5488	1.5423	1.1708	1.5273
7	2.0035	1.8343	1.7504	1.6530	1.3856	1.8343
	7	8	9	10	11	12
1	0.4355	1.2150	0.3979	0.8779	0.7586	0.4951
2	0.8183	1.4201	0.5484	1.0925	1.1366	0.7920
3	0.9752	1.5641	0.7430	1.2084	1.3660	1.0454
4	1.2236	1.6200	0.8403	1.2894	1.4498	1.2889
5	1.3847	1.7261	0.9710	1.3276	1.5070	1.3026
6	1.7461	1.8191	1.0734	1.3786	1.6622	1.6040
7	1.9324	1.8720	1.2884	1.4963	1.7614	1.6817
	13	14	15	16	17	18
1	1.2380	0.5861	0.5484	1.0903	1.3074	0.4355
2	1.4690	0.7430	0.7928	1.1321	1.5286	0.8026
3	1.5423	0.9554	0.9056	1.2515	1.7831	0.7430
4	1.6347	1.0535	1.1237	1.4566	1.8424	1.0269
5	1.6786	1.2836	1.3585	1.4454	1.8569	1.1314
6	1.7633	1.4580	1.4195	1.6105	1.9480	1.3508
7	1.8848	1.6294	1.6511	1.6744	2.0245	1.5123
	19	20	21	22	23	24
1	0.7707	0.5229	1.1087	1.5580	1.2011	1.3160
2	1.0808	0.8957	1.4140	1.6762	1.4412	1.5914
3	1.2115	0.8026	1.6242	1.7124	1.5198	1.6472
4	1.3222	1.1938	1.6632	1.7308	1.6669	1.6675
5	1.4260	1.3957	1.7719	1.8159	1.6996	1.7423
6	1.5198	1.6337	1.8685	1.9233	1.8862	1.8393
7	1.7892	1.7298	1.9757	1.9988	1.9965	1.9805

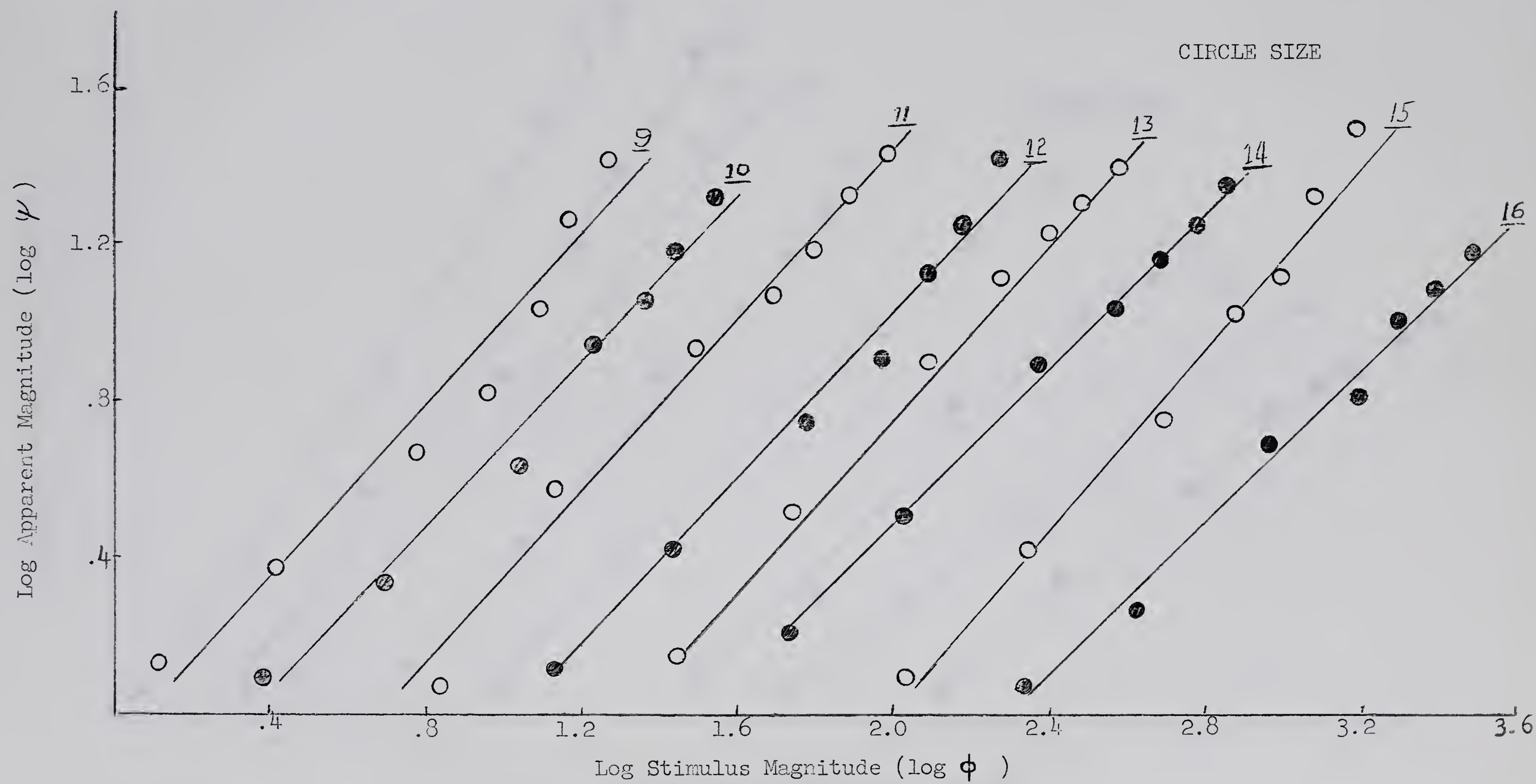
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]

APPENDIX C

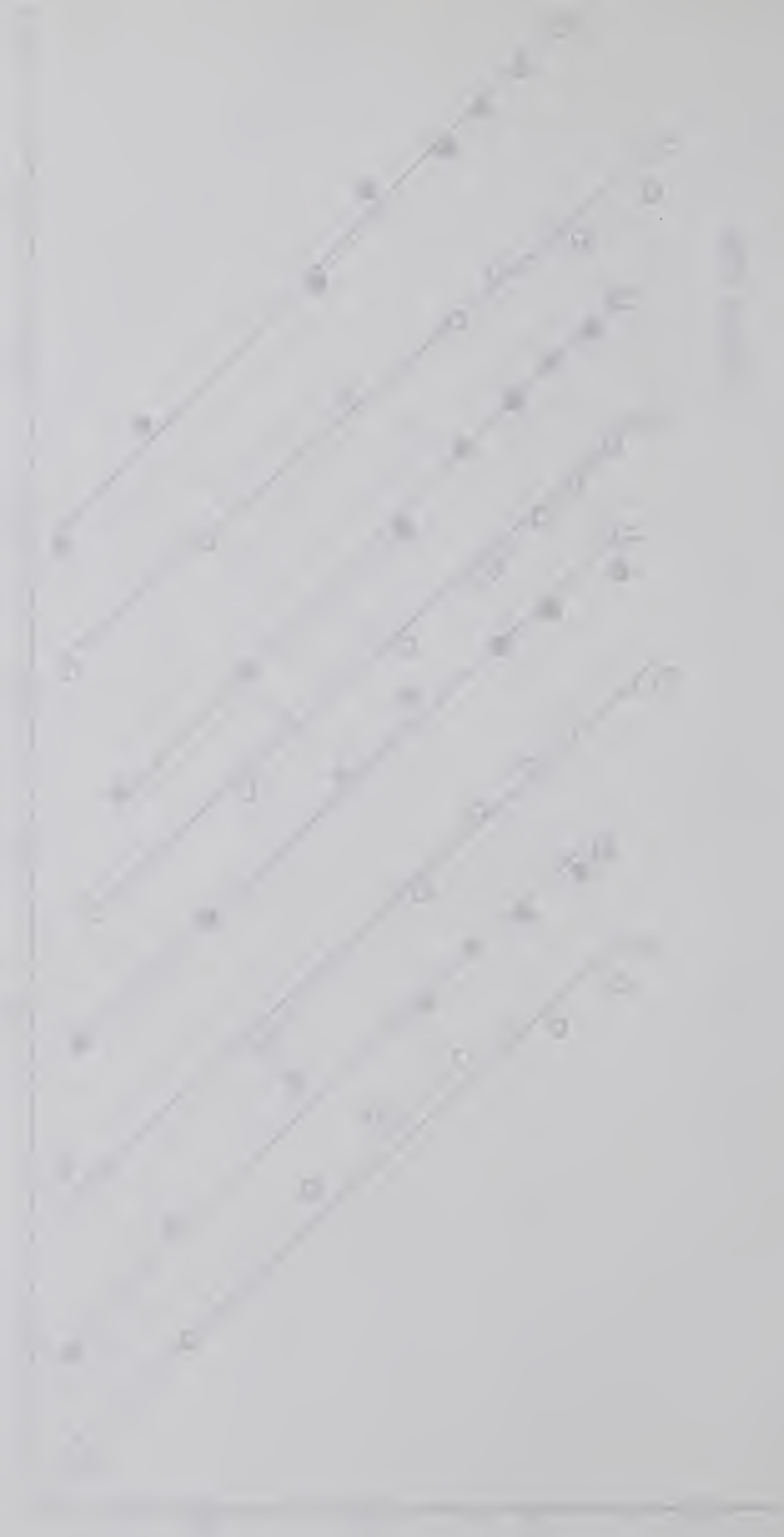
Plots of Individual Psychophysical Functions

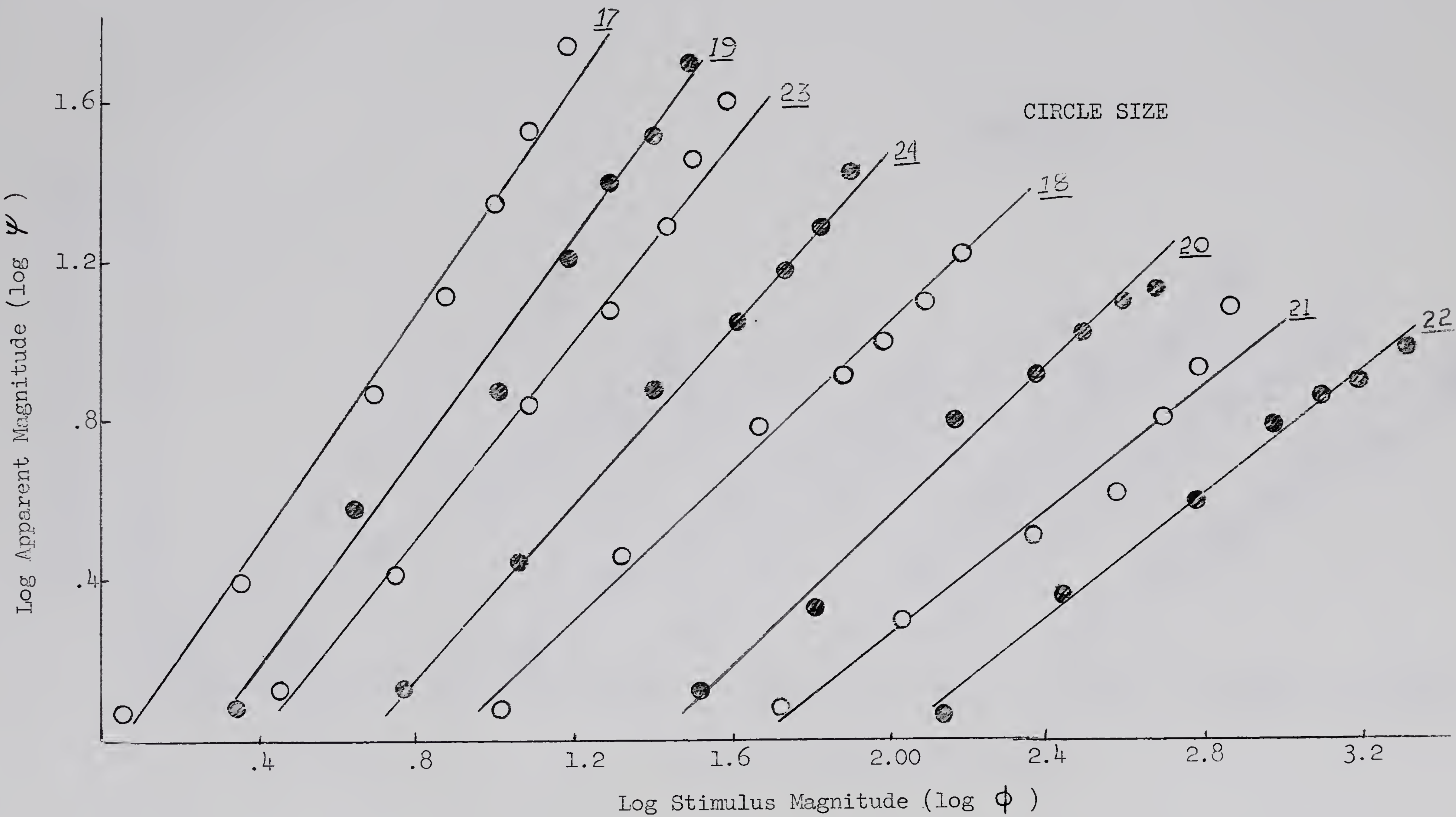


Appendix C1. Individual psychophysical functions for circle size. Each curve is for a single observer. Each point is the mean log of eight responses. Broken lines connect points. Solid lines are the lines of best fit. Relative positions along either axis are arbitrary.



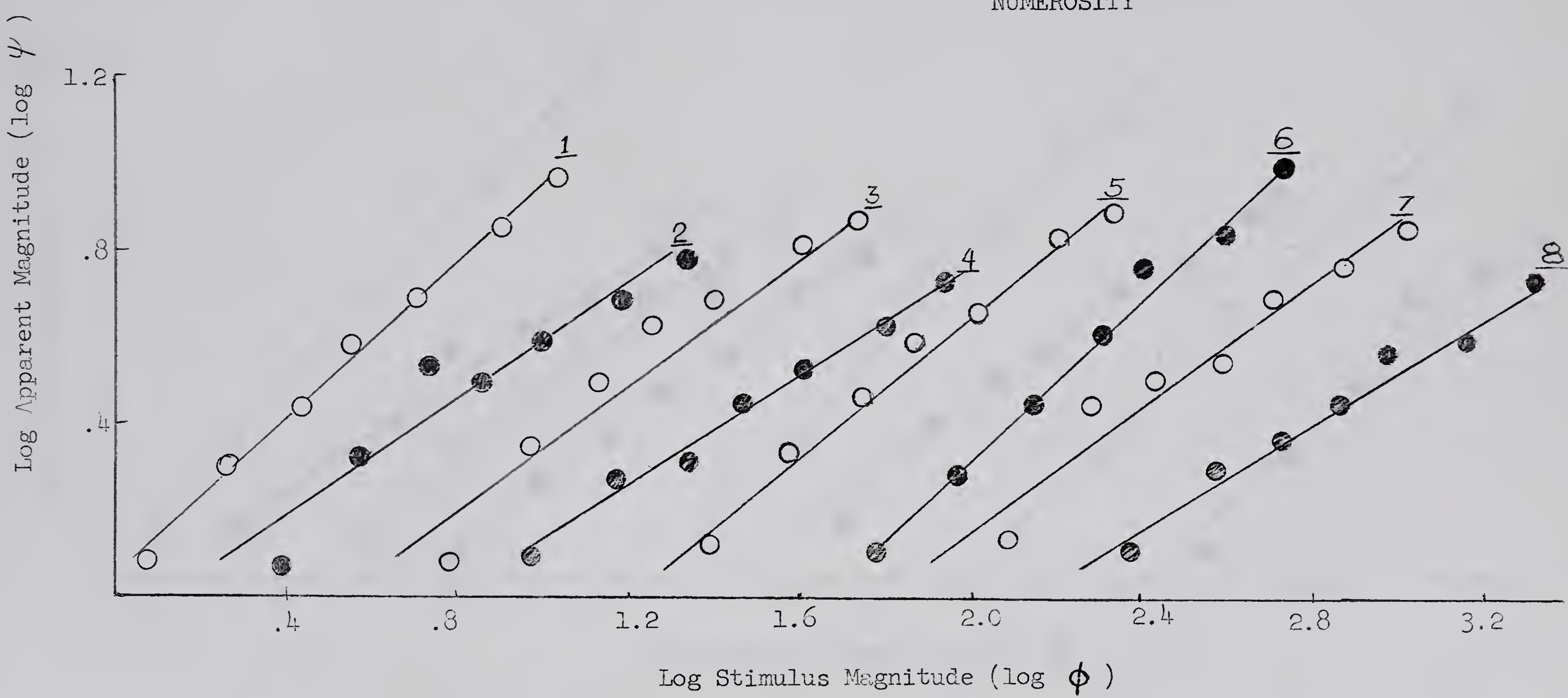
Appendix C1. Individual psychophysical functions for circle size. Each curve is for a single observer. Each point is the mean log of eight responses. Broken lines connect points. Solid lines are the lines of best fit. Relative positions along either axis are arbitrary.



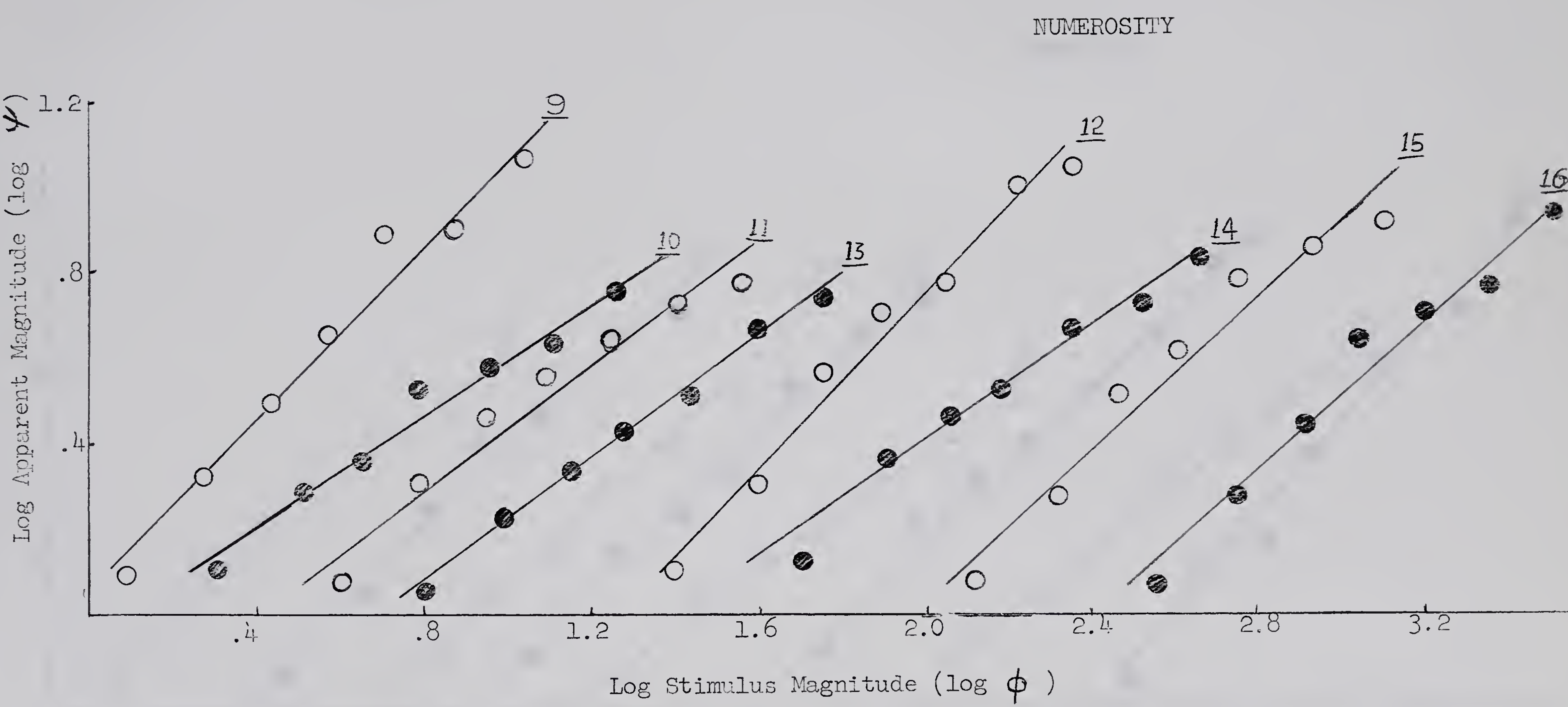


Appendix C1. Individual psychophysical functions for circle size. Each curve is for a single observer. Each point is the mean log of eight responses. Broken lines connect points. Solid lines are the lines of best fit. Relative positions along either axis are arbitrary.

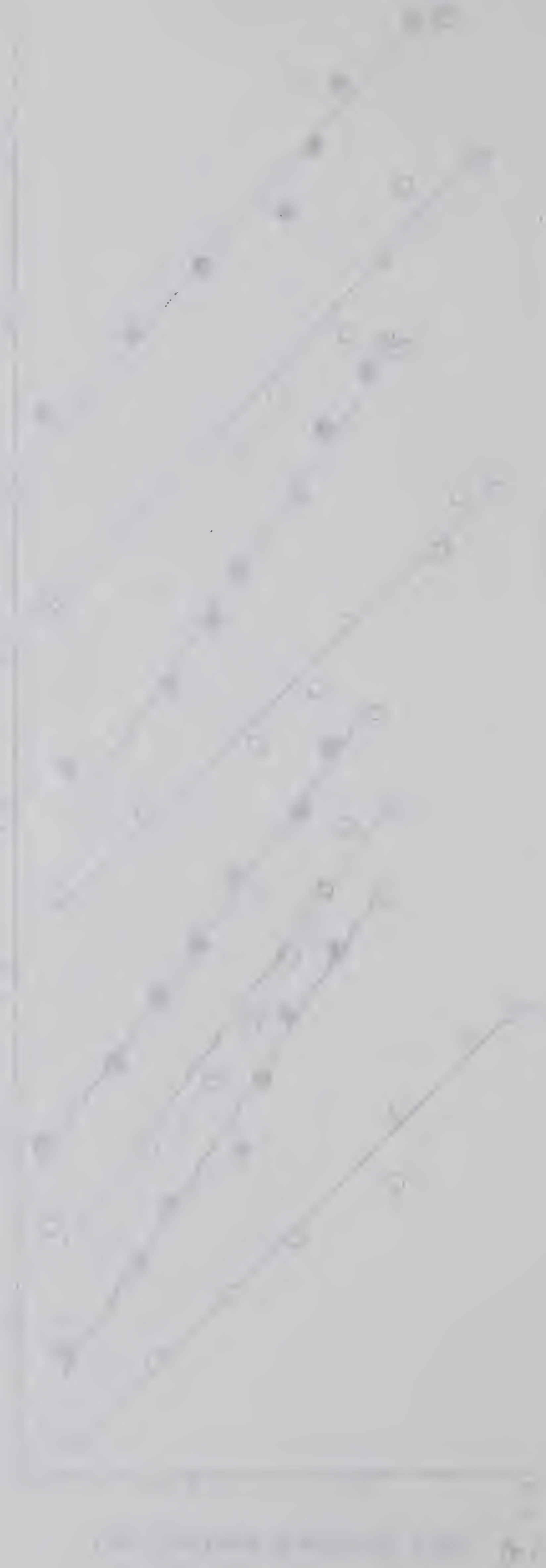
NUMEROSITY



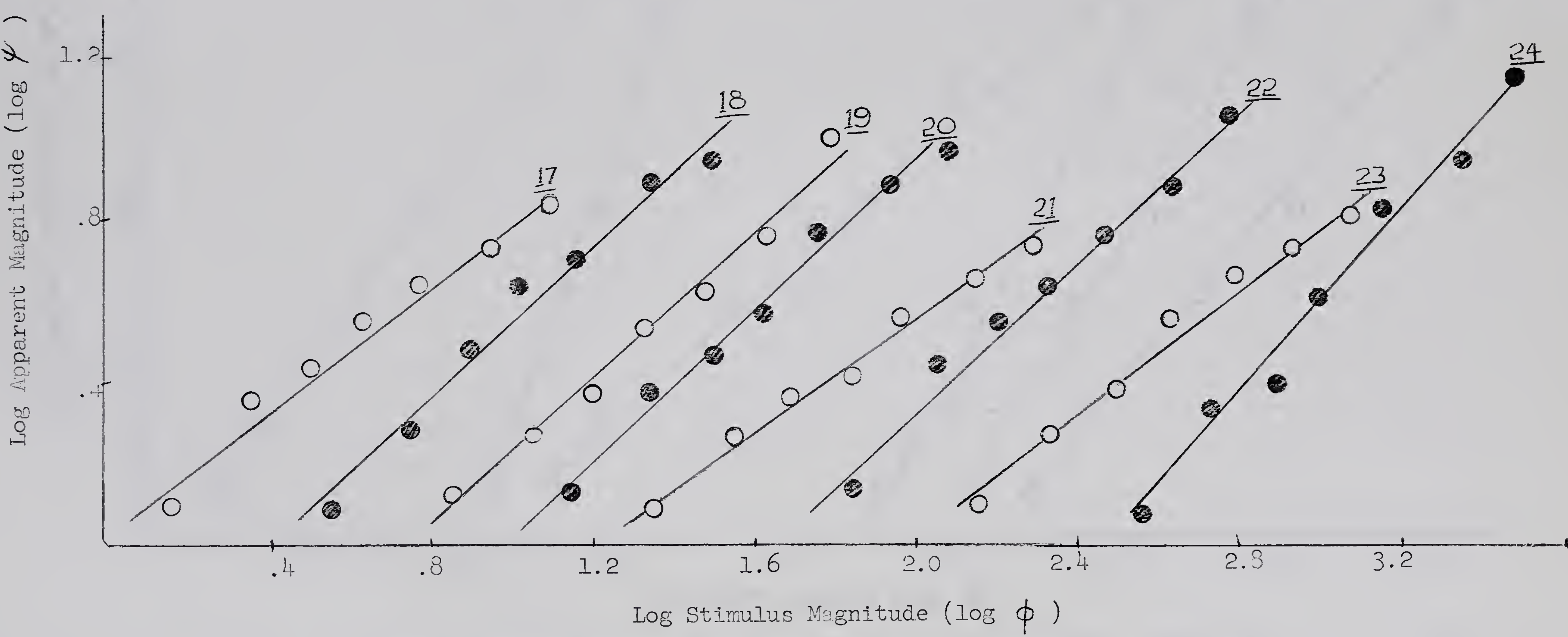
Appendix C2. Individual psychophysical functions for numerosity. Each curve is for a single observer. Each point is the mean log of eight responses. Broken lines connect points. Solid lines are the lines of best fit. Relative positions along either axis are arbitrary.



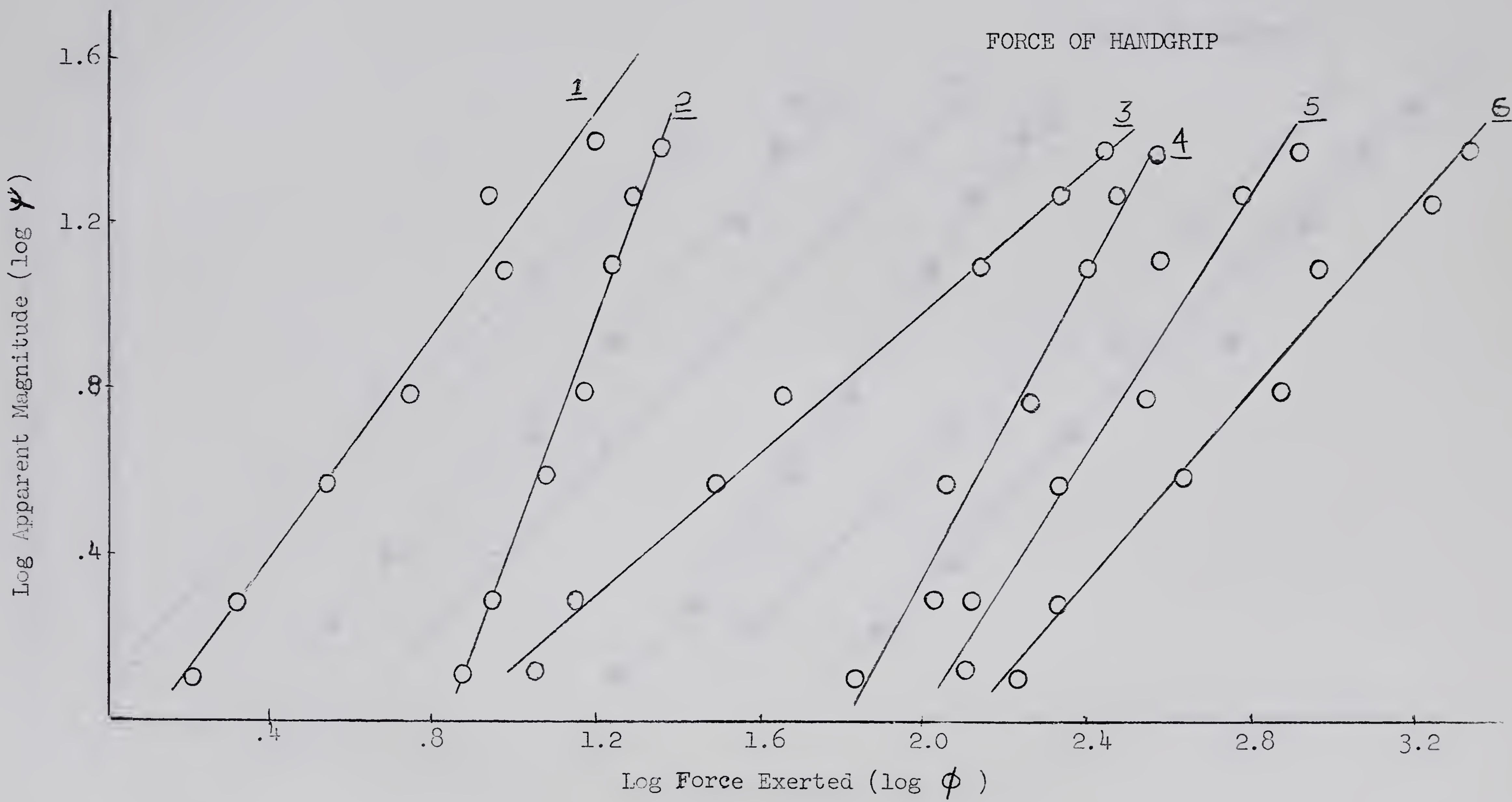
Appendix C2. Individual psychophysical functions for numerosity. Each curve is for a single observer. Each point is the mean log of eight responses. Broken lines connect points. Solid lines are the lines of best fit. Relative positions along either axis are arbitrary.



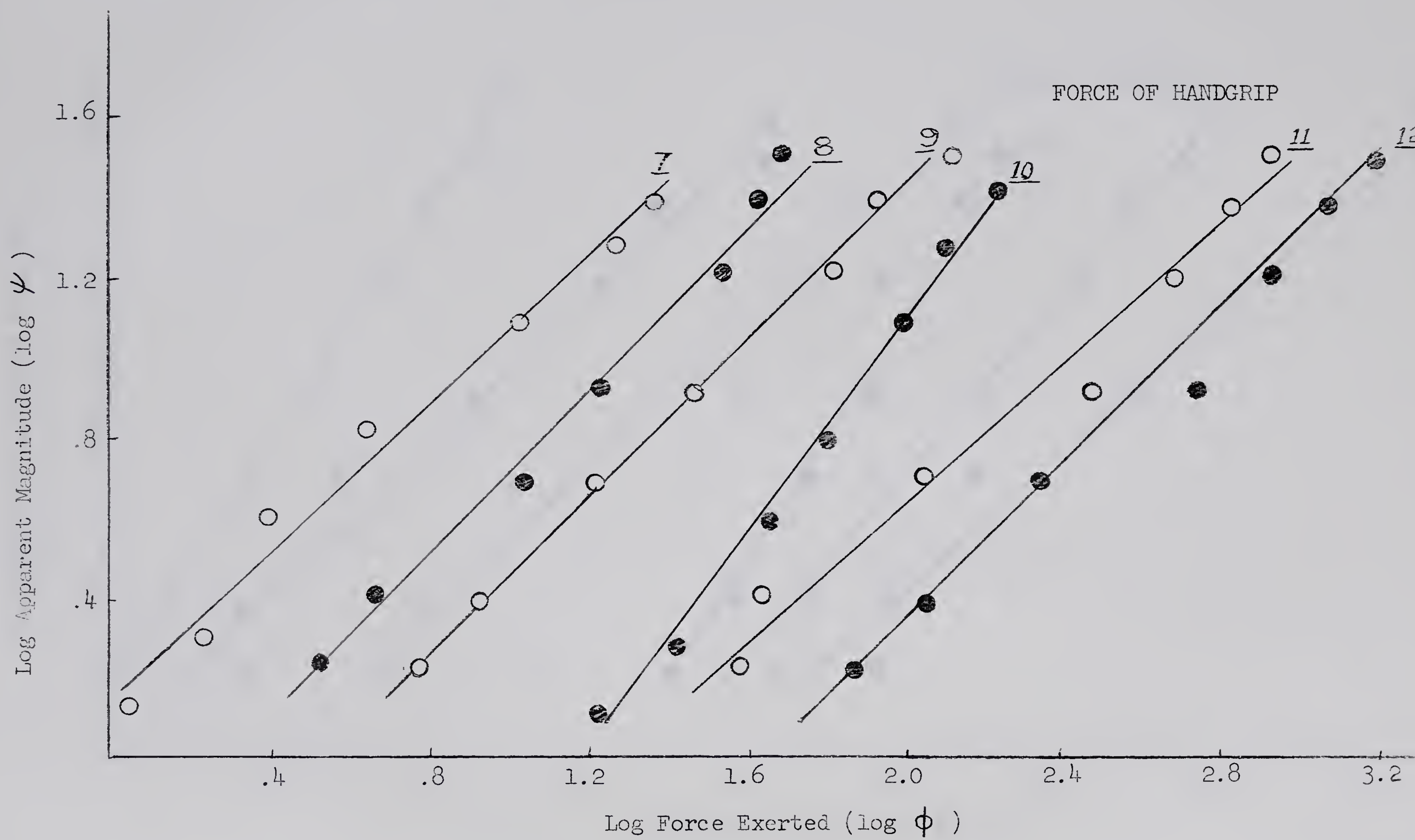
NUMEROSITY



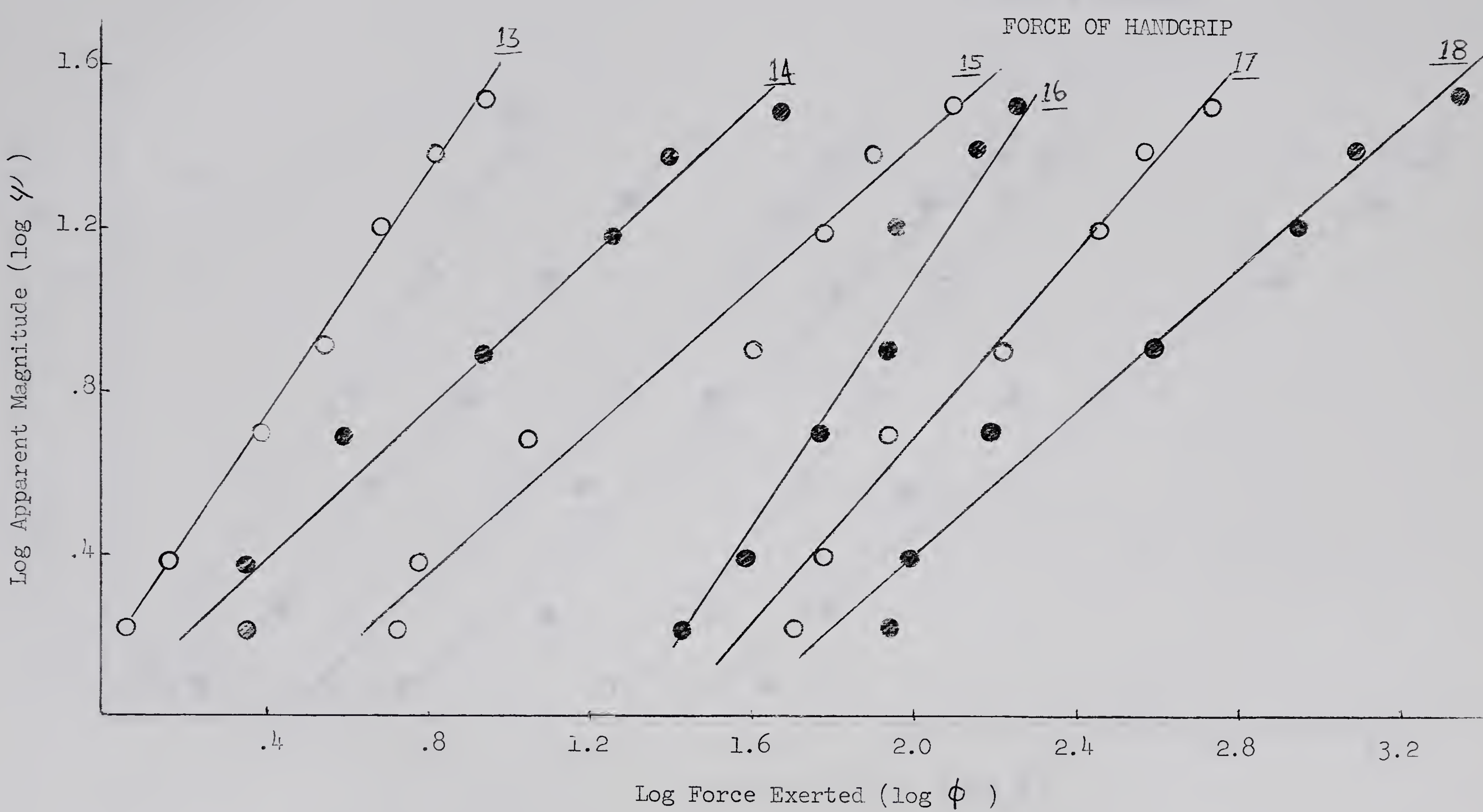
Appendix C2. Individual psychophysical functions for numerosity. Each curve is for a single observer. Each point is the mean log of eight responses. Broken lines connect points. Solid lines are the lines of best fit. Relative positions along either axis are arbitrary.



Appendix C3. Individual psychophysical functions for force of handgrip. Each curve is for a single observer. Each point is the mean log of eight responses. Broken lines connect points. Solid lines are the lines of best fit. Relative positions along either axis are arbitrary.

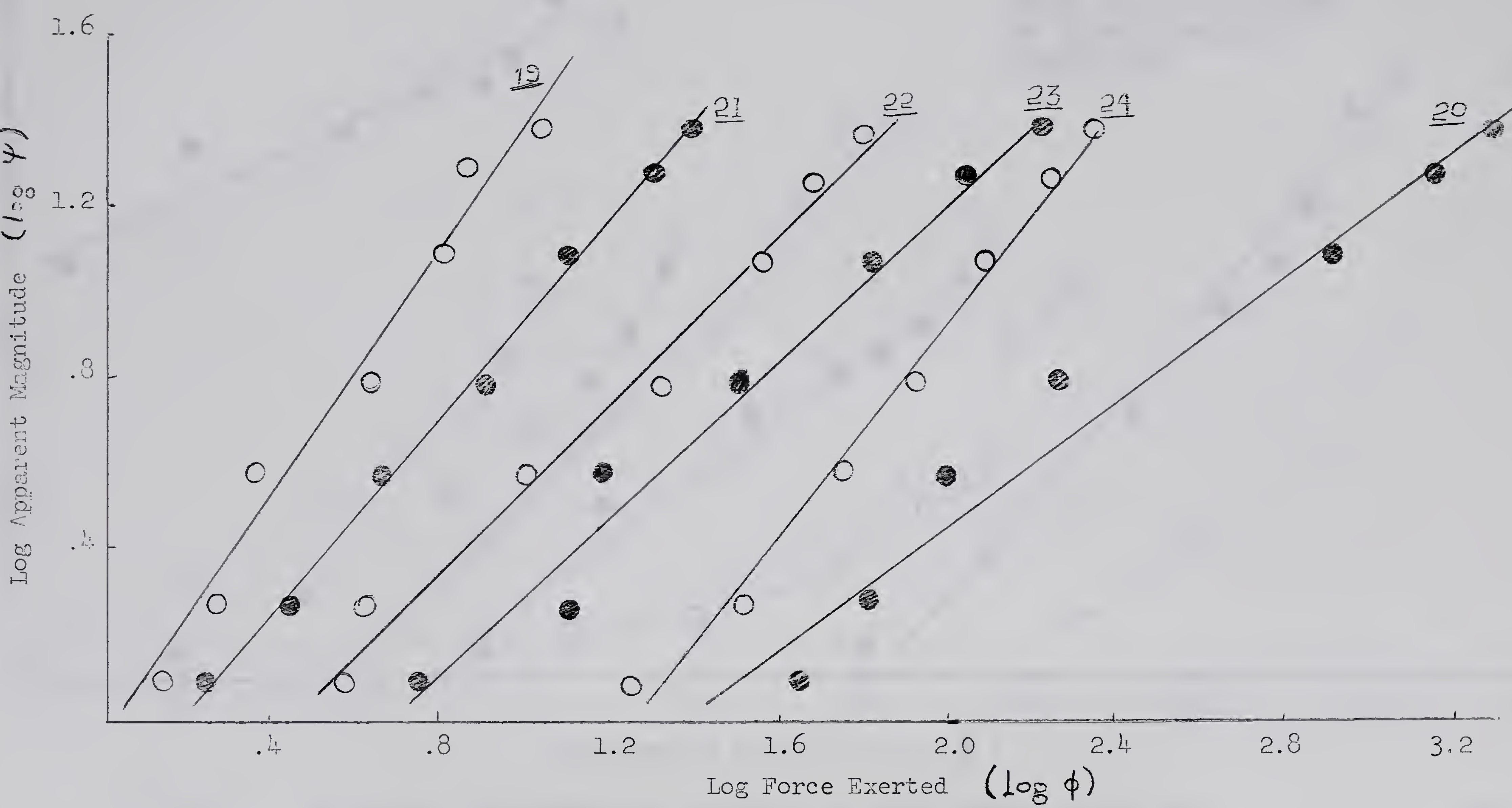


Appendix C3. Individual psychophysical functions for force of handgrip. Each curve is for a single observer. Each point is the mean log of eight responses. Broken lines connect points. Solid lines are the lines of best fit. Relative positions along either axis are arbitrary.

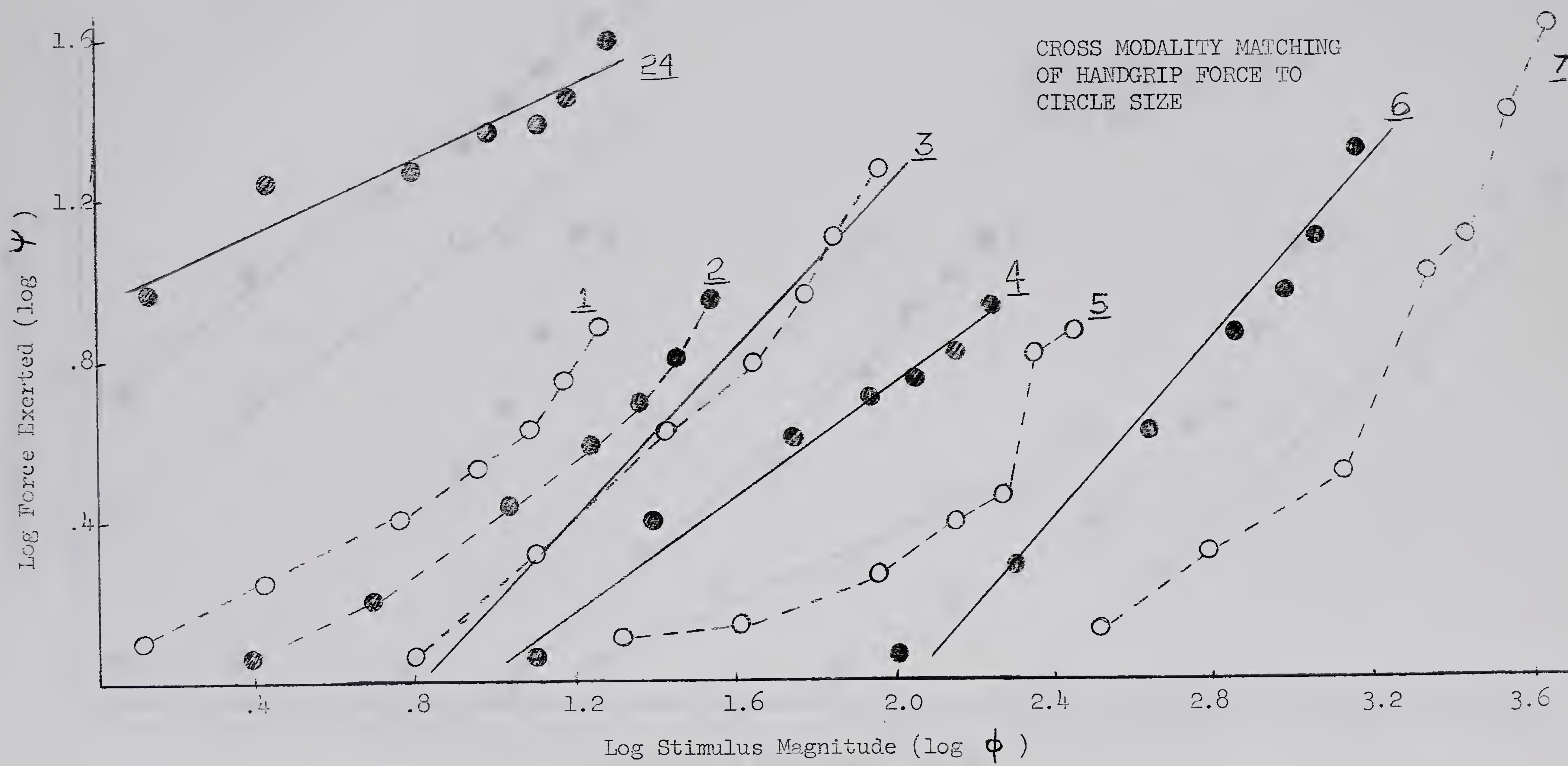


Appendix C3. Individual psychophysical functions for force of handgrip. Each curve is for a single observer. Each point is the mean log of eight responses. Broken lines connect points. Solid lines are the lines of best fit. Relative positions along either axis are arbitrary.

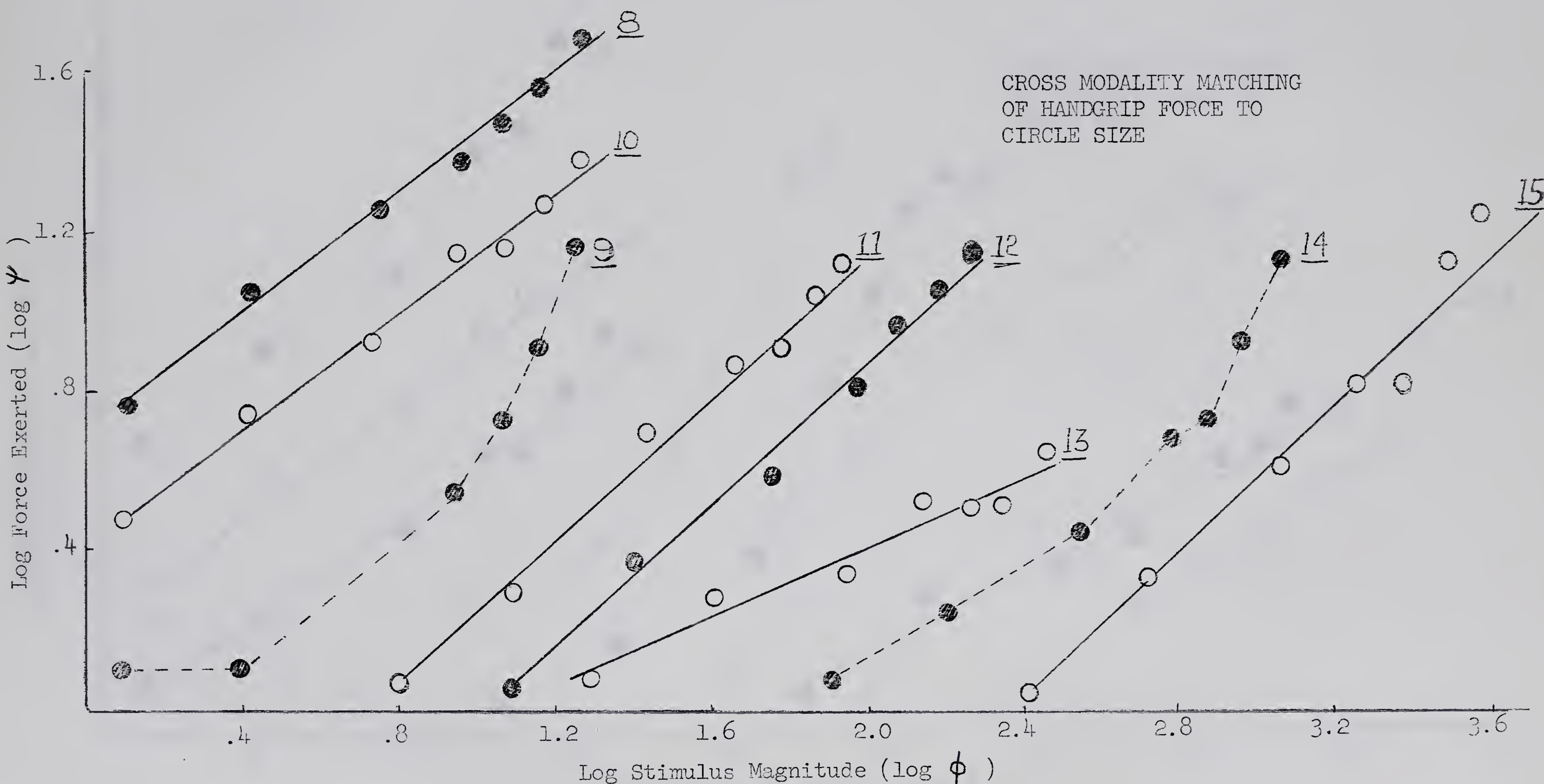
FORCE OF HANDGRIP



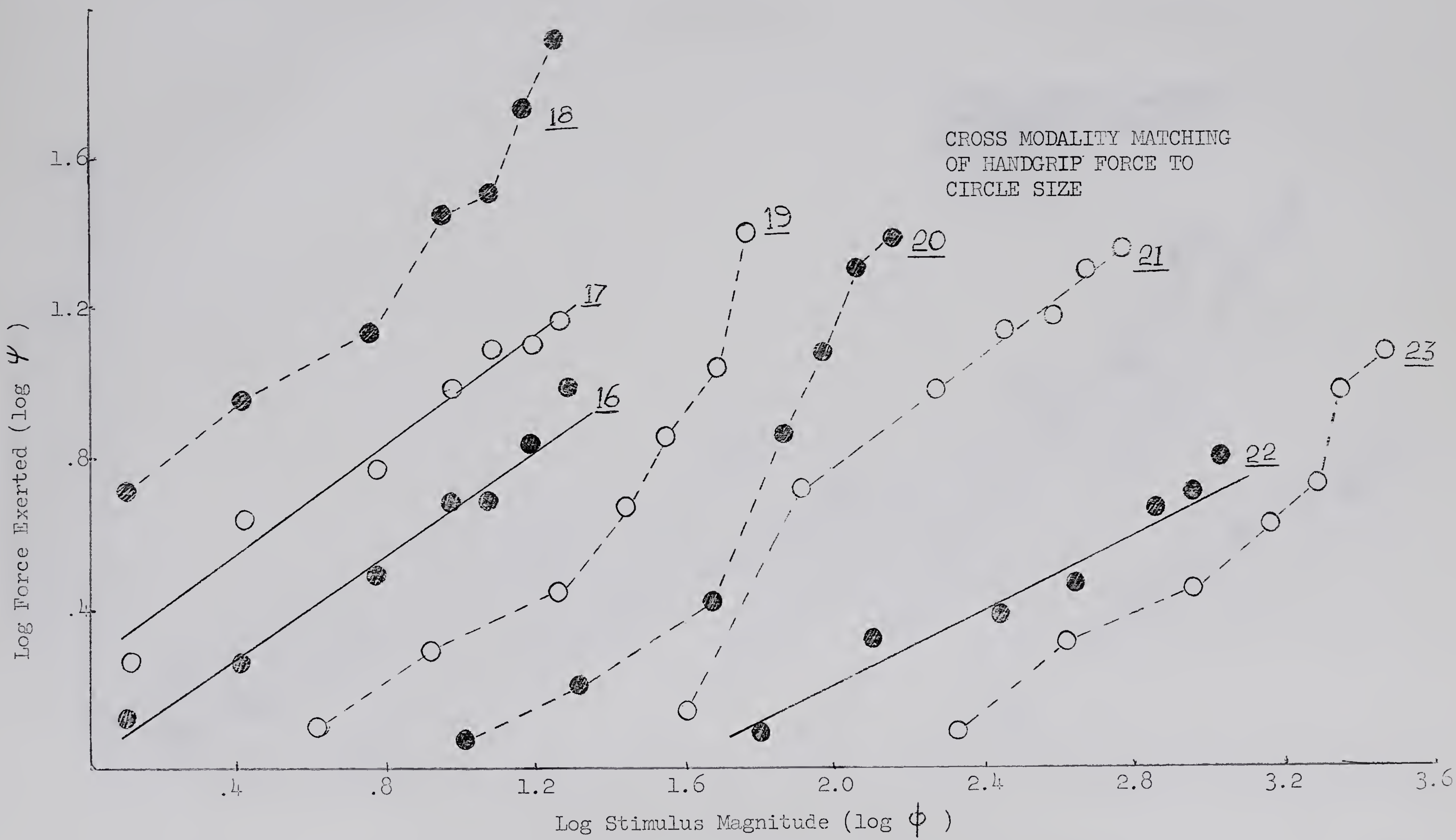
Appendix C3. Individual psychophysical functions for force of handgrip. Each curve is for a single observer. Each point is the mean log of eight responses. Broken lines connect points. Solid lines are the lines of best fit. Relative positions along either axis are arbitrary.



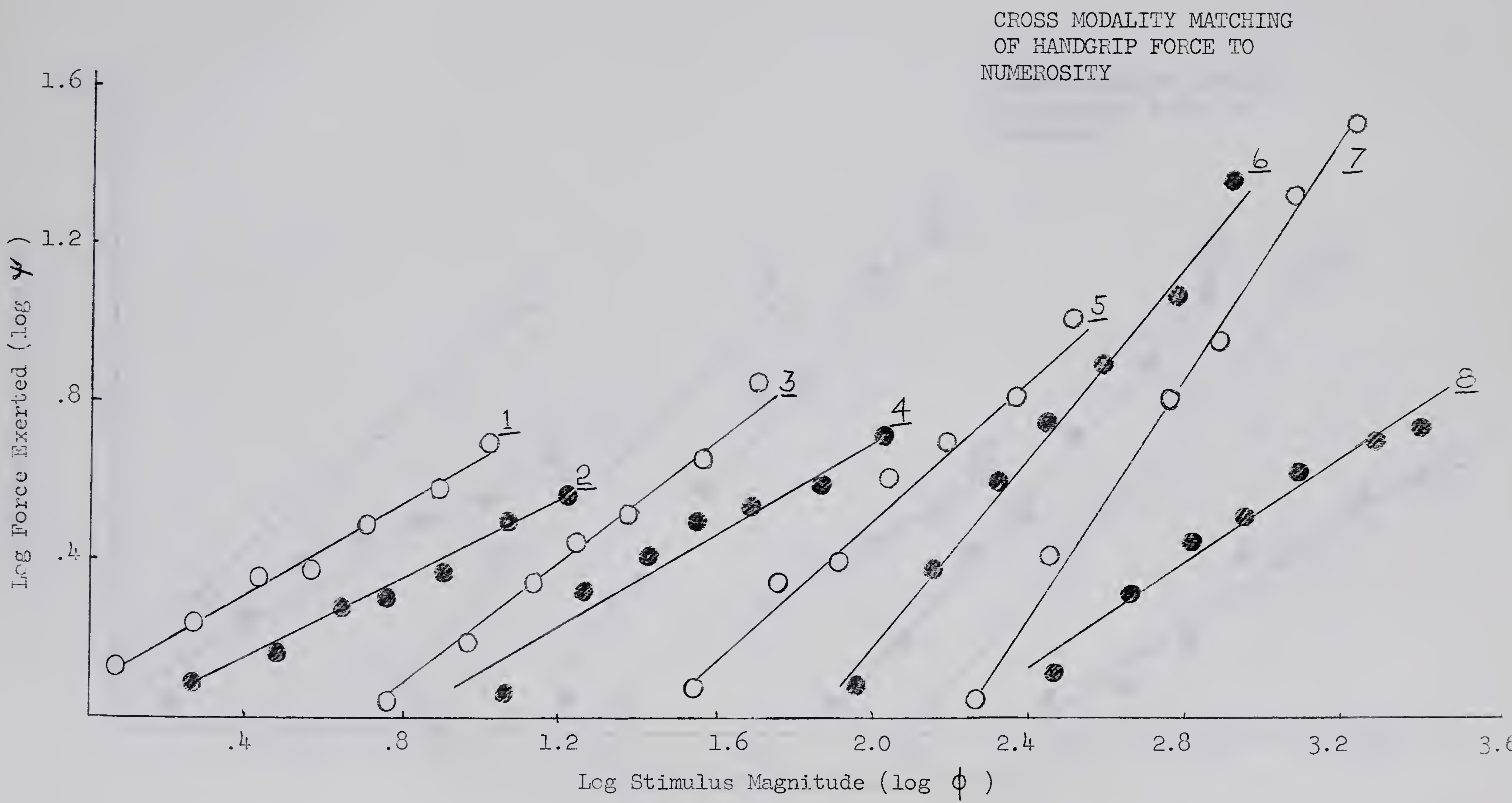
Appendix C4. Individual psychophysical functions for cross modality matching of handgrip force to circle size. Each curve is for a single observer. Each point is the mean log of eight responses. Broken lines connect points. Solid lines are the lines of best fit. Relative positions along either axis are arbitrary.



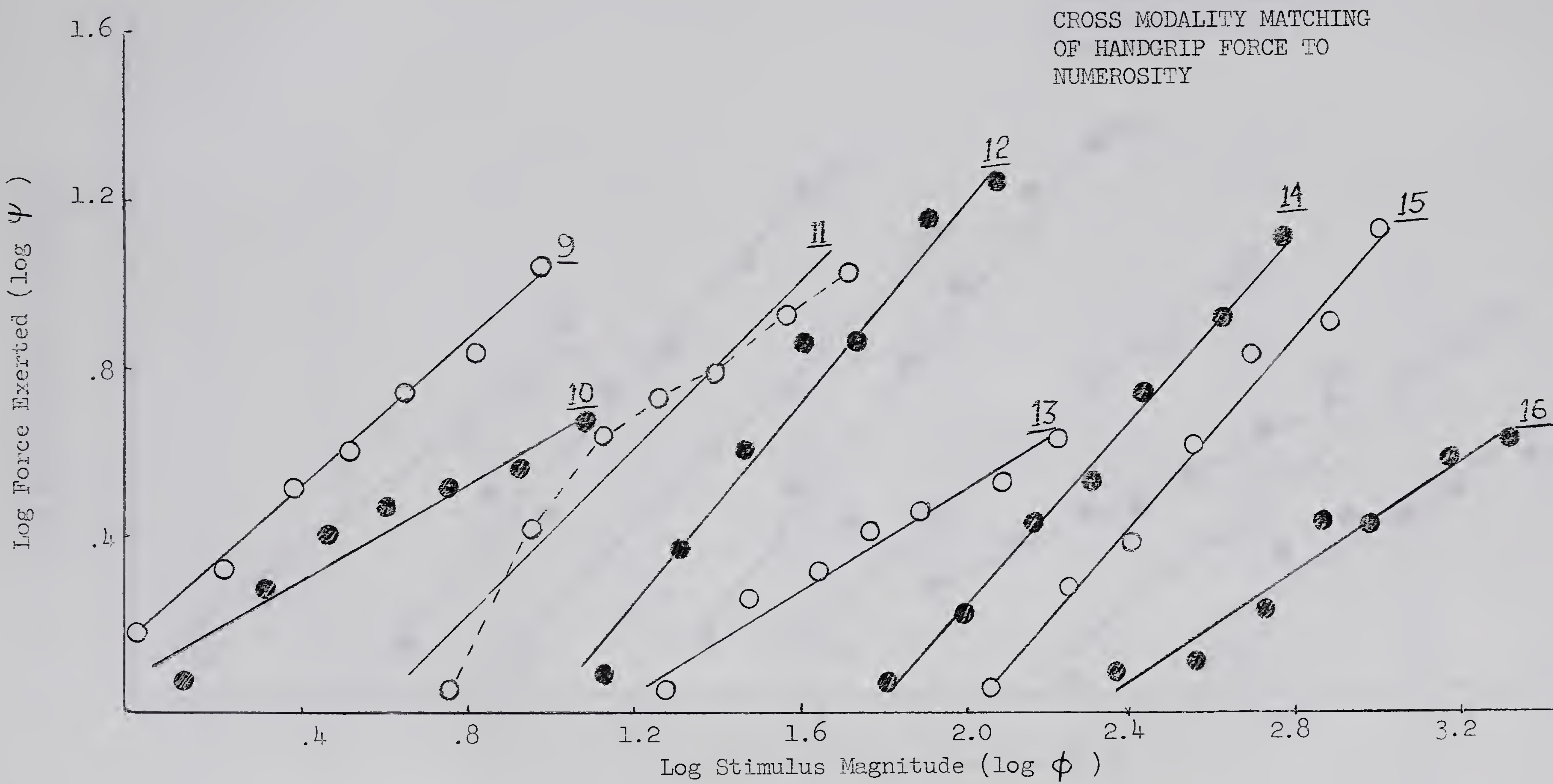
Appendix C4. Individual psychophysical functions for cross modality matching of handgrip force to circle size. Each curve is for a single observer. Each point is the mean log of eight responses. Broken lines connect points. Solid lines are the lines of best fit. Relative positions along either axis are arbitrary.



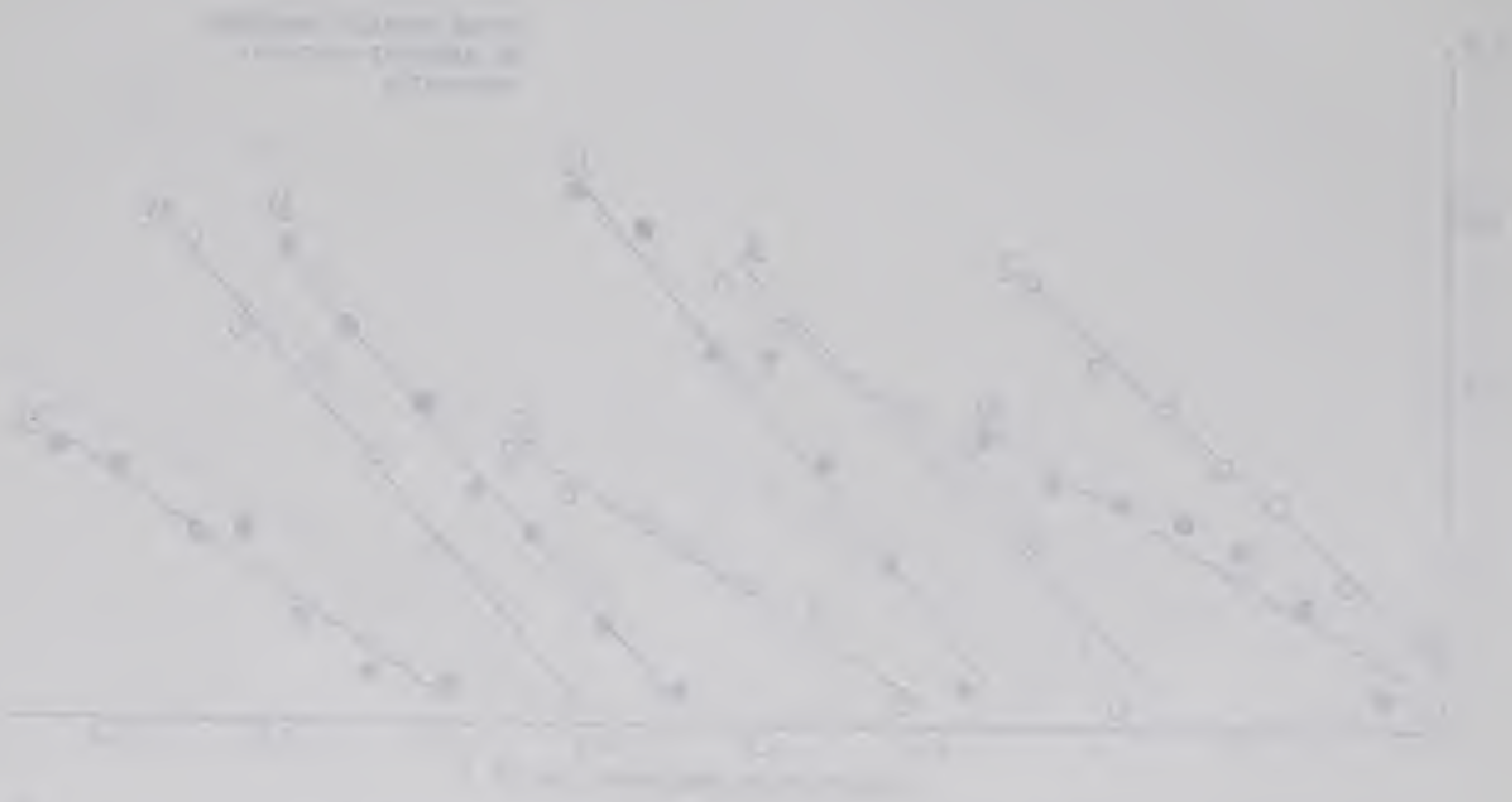
Appendix C4. Individual psychophysical functions for cross modality matching of handgrip force to circle size. Each curve is for a single observer. Each point is the mean log of eight responses. Broken lines connect points. Solid lines are the lines of best fit. Relative positions along either axis are arbitrary.



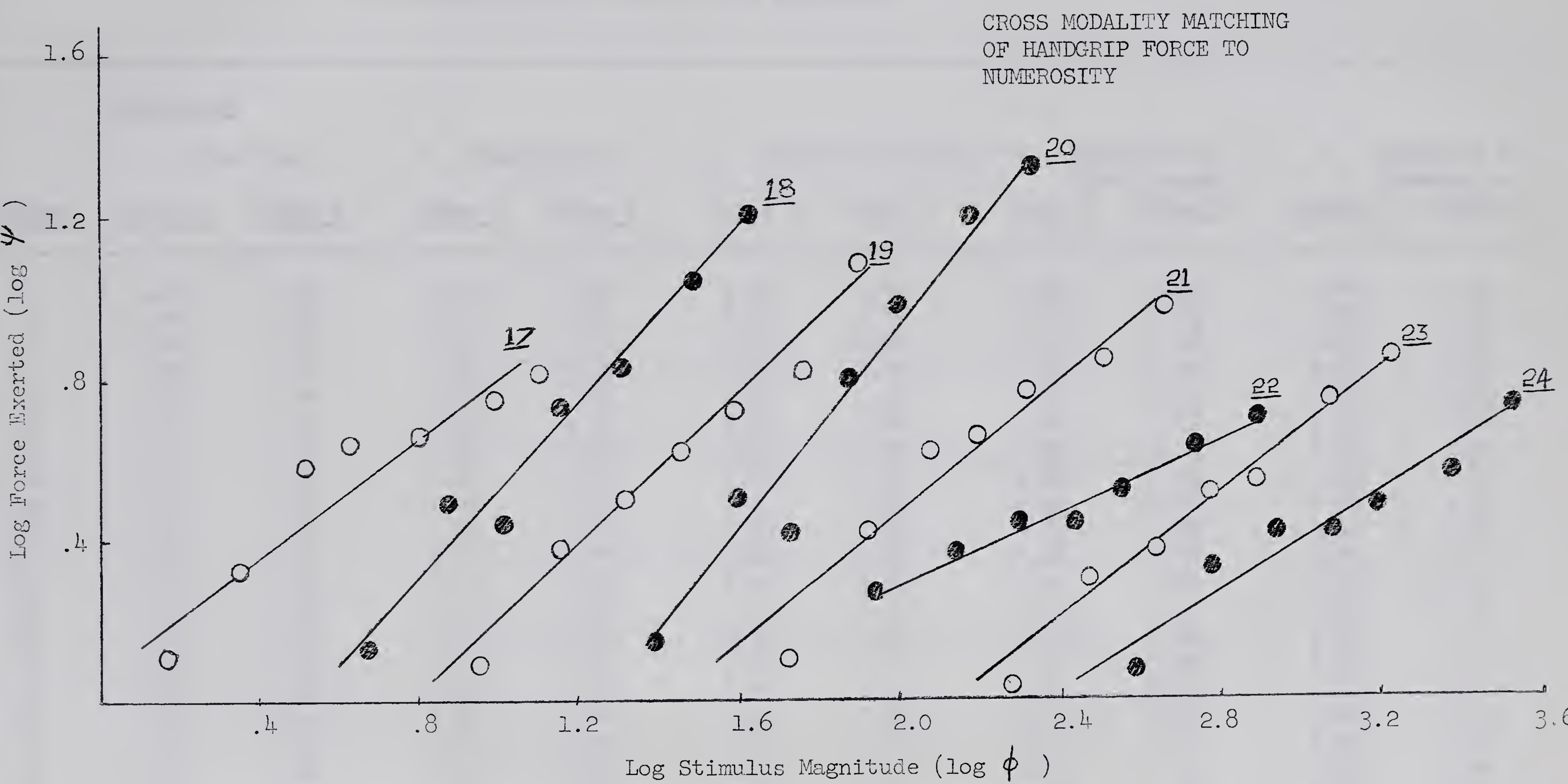
Appendix C5. Individual psychophysical functions for cross modality matching of handgrip force to numerosity. Each curve is for a single observer. Each point is the mean log of eight responses. Broken lines connect points. Solid lines are the lines of best fit. Relative positions along either axis are arbitrary.



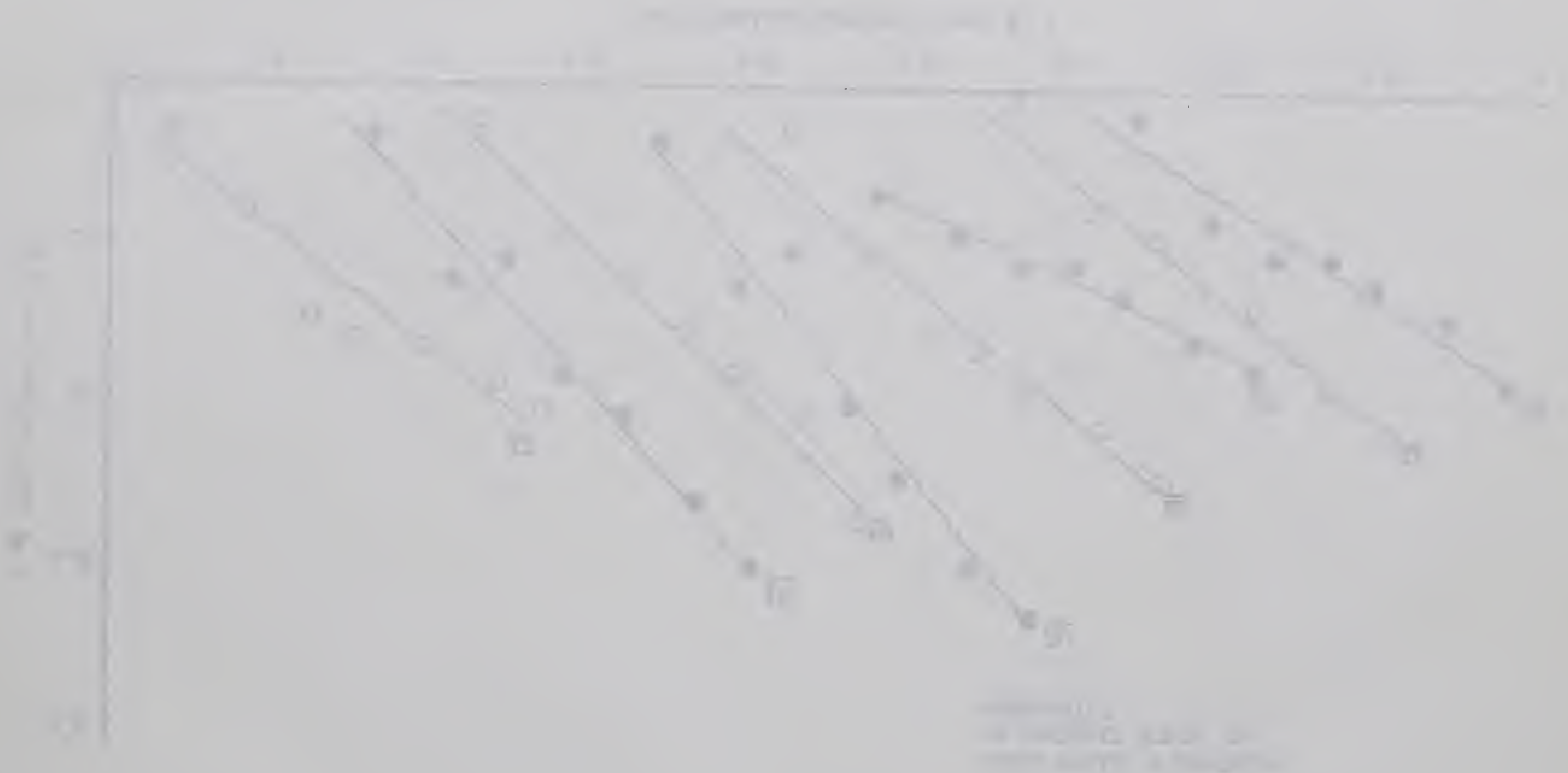
Appendix C5. Individual psychophysical functions for cross modality matching of handgrip force to numerosity. Each curve is for a single observer. Each point is the mean log of eight responses. Broken lines connect points. Solid lines are the lines of best fit. Relative positions along either axis are arbitrary.



[illegible]



Appendix C5. Individual psychophysical functions for cross modality matching of handgrip force to numerosity. Each curve is for a single observer. Each point is the mean log of eight responses. Broken lines connect points. Solid lines are the lines of best fit. Relative positions along either axis are arbitrary.



APPENDIX D_1

Individual Order-1 and Order-2 Exponents

Subject	Condition: 1. Circle Size		2. Numerosity		3. Force of Handgrip		4. Handgrip to Circle Size		5. Handgrip to Numerosity	
	Order 1	Order 2	Order 1	Order 2	Order 1	Order 2	Order 1	Order 2	Order 1	Order 2
1	1.16	1.35	0.91	0.97	1.29	1.40	0.62	0.68	0.69	0.50
2	1.07	1.39	0.53	0.88	2.95	2.59	0.74	0.79	0.37	0.63
3	1.20	1.01	0.79	0.79	0.92	0.87	0.98	1.08	0.76	0.87
4	1.85	1.48	0.67	0.60	2.11	1.59	0.62	0.79	0.66	0.55
5	1.21	1.37	0.80	0.84	1.54	1.48	0.56	0.65	0.89	1.00
6	1.19	1.25	0.99	0.89	1.25	1.06	1.10	1.13	1.17	1.38
7	1.19	1.29	0.65	0.76	0.92	0.81	1.28	1.26	1.59	1.30
8	0.70	0.70	0.65	0.58	0.97	1.12	0.82	0.82	0.63	0.73
9	1.11	1.06	1.04	0.97	0.97	0.94	0.97	0.86	0.75	1.05
10	0.86	1.28	0.70	0.63	1.43	1.27	0.69	0.80	0.54	0.63
11	1.13	1.13	0.77	0.73	0.81	0.96	0.84	1.08	1.02	0.92
12	1.07	1.07	1.02	1.07	1.05	0.89	0.96	0.98	1.33	1.16
13	1.11	1.08	0.73	0.72	1.33	1.70	0.47	0.47	0.57	0.65
14	1.17	0.77	0.67	0.74	0.93	0.96	0.78	0.97	1.00	1.23
15	1.19	1.14	0.92	0.94	1.00	0.74	1.07	0.93	1.18	1.10
16	0.91	1.06	0.96	0.85	1.88	1.55	0.71	0.72	0.61	0.70
17	1.42	1.41	0.71	0.77	1.01	1.40	0.58	0.95	0.62	0.79
18	0.99	0.92	0.89	0.94	0.85	0.85	0.95	1.04	1.06	1.10
19	1.49	1.24	0.91	0.90	1.57	1.36	1.02	1.03	0.94	0.96
20	0.93	0.92	0.88	0.89	0.78	0.70	1.18	1.31	1.24	1.35
21	0.82	0.82	0.64	0.72	0.92	1.53	1.16	0.80	0.70	0.98
22	0.83	0.76	0.75	1.04	0.81	1.27	0.58	0.46	0.46	0.42
23	1.42	1.18	0.77	0.75	0.85	0.96	0.79	0.87	0.81	0.77
24	1.11	1.13	1.07	1.19	1.18	1.32	0.42	0.52	0.64	0.55

[illegible]

[illegible]

[illegible]

Individual Exponents for each Trial

Circle Size								
Subject	Trial 1	2	3	4	5	6	7	8
1	1.23	1.18	1.16	1.09	1.32	1.39	1.33	1.36
2	0.92	1.04	1.23	1.10	1.33	1.38	1.45	1.40
3	1.22	1.22	1.20	1.18	1.02	1.01	1.10	0.94
4	1.27	2.38	1.56	1.86	1.37	1.54	1.49	1.50
5	0.99	1.19	1.33	1.34	1.26	1.40	1.39	1.44
6	1.18	1.20	1.21	1.20	1.22	1.28	1.24	1.27
7	1.08	1.26	1.20	1.22	1.29	1.25	1.28	1.34
8	0.65	0.76	0.74	0.66	0.67	0.65	0.68	0.79
9	1.08	1.09	1.13	1.12	1.13	1.06	1.06	1.13
10	0.90	0.87	0.87	0.80	1.30	1.23	1.30	1.30
11	1.16	1.22	1.15	0.99	1.18	1.19	1.11	1.07
12	1.08	1.09	1.10	1.04	1.00	1.06	1.08	1.16
13	1.13	1.13	1.06	1.13	1.08	1.06	1.06	1.14
14	1.16	1.13	1.19	1.23	0.89	0.76	0.72	0.72
15	1.18	1.12	1.22	1.23	1.14	1.06	1.18	1.17
16	0.87	0.96	1.00	0.80	1.10	1.02	1.19	0.93
17	1.37	1.38	1.58	1.34	1.32	1.39	1.48	1.45
18	0.87	0.90	1.08	1.10	0.82	0.89	0.97	1.01
19	0.92	1.30	1.88	1.85	1.28	1.16	1.26	1.27
20	0.87	0.92	0.97	0.97	0.91	0.90	0.93	0.95
21	0.81	0.85	0.88	0.72	0.91	0.76	0.80	0.82
22	0.75	0.83	0.94	0.82	0.94	0.89	0.68	0.57
23	1.32	1.48	1.41	1.46	1.23	1.06	1.21	1.21
24	1.06	1.17	1.10	1.11	1.12	1.15	1.14	1.13

[illegible]

[illegible]								
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible

Numerosity

Subject	Trial 1	2	3	4	5	6	7	8
1	0.90	0.89	0.89	0.88	1.00	0.98	0.95	0.94
2	0.09	0.38	0.86	0.76	0.78	0.92	0.88	0.95
3	0.76	0.82	0.86	0.70	0.78	0.79	0.84	0.76
4	0.67	0.69	0.73	0.59	0.71	0.59	0.50	0.59
5	0.77	0.78	0.89	0.78	0.88	0.80	0.85	0.82
6	0.81	1.09	1.06	0.98	0.82	0.87	0.92	0.94
7	0.66	0.71	0.60	0.62	0.79	0.84	0.65	0.75
8	0.68	0.63	0.65	0.63	0.58	0.52	0.59	0.63
9	1.03	1.09	1.00	1.05	1.05	1.02	0.98	0.82
10	0.79	0.59	0.74	0.69	0.72	0.60	0.59	0.61
11	0.71	0.89	0.74	0.73	0.74	0.73	0.70	0.74
12	0.92	1.07	1.05	1.02	1.04	1.11	1.10	1.04
13	0.65	0.72	0.81	0.72	0.71	0.73	0.67	0.74
14	0.72	0.62	0.67	0.66	0.78	0.77	0.68	0.71
15	0.90	1.08	0.97	0.75	0.99	0.84	1.09	0.84
16	1.01	0.99	1.00	0.84	0.87	0.94	0.73	0.86
17	0.66	0.76	0.73	0.70	0.77	0.79	0.75	0.75
18	0.78	0.92	0.97	0.88	1.04	0.97	0.94	0.81
19	0.69	0.91	1.03	1.00	0.94	0.89	0.79	0.97
20	0.78	0.86	0.96	0.93	0.91	0.89	0.87	0.90
21	0.68	0.58	0.65	0.66	0.80	0.67	0.77	0.66
22	0.68	0.80	0.69	0.83	1.04	1.15	0.95	1.05
23	0.85	0.71	0.78	0.74	0.77	0.74	0.73	0.75
24	0.94	1.10	1.09	1.15	1.24	1.21	1.07	1.21

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Force of Handgrip

Subject	Trial 1	2	3	4	5	6	7	8
1	1.00	1.20	1.31	0.63	1.16	1.32	1.56	1.54
2	2.47	2.99	3.12	2.18	2.51	2.16	1.94	3.59
3	1.01	1.06	0.75	0.85	0.83	0.78	1.00	0.79
4	2.07	2.09	2.32	1.74	1.64	1.45	1.70	1.38
5	1.38	1.20	1.57	1.79	1.25	1.27	1.47	1.62
6	1.09	1.23	1.24	1.34	1.27	0.95	0.88	1.14
7	0.78	1.04	0.94	0.85	0.86	0.74	0.78	0.84
8	0.84	0.90	1.14	0.96	0.95	1.04	1.25	1.13
9	0.94	0.98	0.96	0.96	0.92	0.95	0.85	0.92
10	1.52	1.30	1.45	1.36	0.97	1.58	1.38	1.07
11	0.87	0.72	0.78	0.80	1.08	0.94	0.84	0.96
12	0.92	0.94	1.13	0.88	0.94	0.84	0.90	0.81
13	1.06	1.24	1.10	1.68	1.85	1.67	1.64	1.48
14	0.86	0.96	0.91	0.90	1.10	0.96	0.81	0.94
15	0.93	0.98	0.90	0.85	0.69	0.74	0.69	0.75
16	1.29	1.51	2.40	1.83	1.14	1.66	1.27	1.90
17	0.82	0.88	0.93	1.34	1.27	1.44	1.36	1.35
18	0.94	0.78	0.83	0.81	0.89	0.84	0.77	0.86
19	1.00	1.30	1.63	2.12	1.20	1.15	1.34	1.49
20	0.69	0.81	0.73	0.82	0.64	0.73	0.66	0.66
21	0.78	0.92	0.91	1.04	1.26	1.50	1.54	1.81
22	0.70	0.82	0.86	0.76	0.86	1.28	1.18	1.57
23	0.70	0.80	0.82	0.97	0.80	0.99	0.86	1.16
24	0.94	1.01	1.17	1.34	1.13	1.22	1.16	1.66

CMM Circle Size

Subject	Trial 1	2	3	4	5	6	7	8
1	0.63	0.53	0.60	0.72	0.63	0.59	0.72	0.77
2	0.69	0.78	0.68	0.79	0.72	0.88	0.70	0.87
3	1.00	0.96	0.88	1.08	1.10	1.00	1.14	1.06
4	0.70	0.41	0.75	0.62	0.61	0.97	0.81	0.77
5	0.60	0.64	0.39	0.59	0.66	0.58	0.68	0.71
6	0.97	0.89	1.08	1.10	1.11	1.18	1.14	1.08
7	1.22	1.32	1.24	1.35	1.28	1.27	1.10	1.38
8	0.52	0.83	1.01	0.57	0.73	0.76	1.04	0.72
9	0.98	0.99	0.96	0.95	0.82	0.78	0.84	0.94
10	0.84	0.72	0.68	0.53	0.89	0.72	0.84	0.77
11	0.58	1.00	1.14	0.64	0.86	0.93	1.24	1.28
12	1.13	0.91	0.98	0.84	1.09	0.89	1.06	0.87
13	0.37	0.46	0.62	0.43	0.45	0.39	0.51	0.53
14	0.65	0.64	0.96	0.86	0.96	1.05	0.94	0.95
15	1.09	1.25	0.87	1.08	1.00	0.89	1.04	0.79
16	0.58	0.72	0.73	0.80	0.98	0.68	0.65	0.58
17	0.75	0.43	0.59	0.55	0.81	0.88	1.39	0.72
18	1.03	0.99	1.06	0.74	0.91	1.00	1.18	1.08
19	0.85	0.99	1.14	1.10	0.90	1.14	1.11	0.97
20	0.99	1.39	1.42	0.90	1.34	1.28	1.30	1.31
21	0.90	1.04	1.25	1.44	0.77	0.77	0.93	0.73
22	0.65	0.42	0.59	0.64	0.49	0.45	0.48	0.42
23	0.64	0.70	0.82	1.00	0.98	0.78	0.76	0.95
24	0.54	0.36	0.40	0.39	0.41	0.58	0.61	0.47

CMM Numerosity

Subject	Trial 1	2	3	4	5	6	7	8
1	0.69	0.72	0.70	0.57	0.50	0.48	0.52	0.50
2	0.19	0.36	0.30	0.65	0.46	0.70	0.69	0.66
3	0.84	0.69	0.80	0.71	1.01	0.76	0.89	0.82
4	0.76	0.55	0.72	0.59	0.72	0.40	0.45	0.64
5	0.78	0.95	0.93	0.89	0.89	0.94	1.15	1.00
6	0.93	1.18	1.56	1.00	1.67	1.16	1.32	1.39
7	1.49	1.72	1.59	1.56	1.58	1.43	1.57	1.43
8	0.54	0.55	0.73	0.62	0.84	0.51	0.90	0.67
9	0.74	0.74	0.82	0.68	0.93	1.15	1.17	0.96
10	0.56	0.58	0.54	0.47	0.58	0.63	0.77	0.54
11	1.04	0.95	1.16	0.96	0.95	0.75	0.92	1.04
12	0.92	1.43	1.51	1.46	1.28	0.93	1.30	1.14
13	0.72	0.52	0.57	0.47	0.65	0.66	0.58	0.73
14	0.97	0.96	0.84	1.22	1.51	1.10	1.01	1.30
15	0.90	1.50	1.39	0.94	1.35	0.90	0.76	1.39
16	0.78	0.79	0.27	0.61	0.72	0.67	0.72	0.72
17	0.64	0.52	0.75	0.58	1.16	0.77	0.54	0.68
18	1.08	1.11	1.03	1.01	1.21	0.94	1.28	0.98
19	1.05	0.70	0.74	1.29	1.08	0.81	0.88	1.05
20	1.64	0.68	1.34	1.30	1.27	1.17	1.51	1.47
21	0.71	0.70	0.70	0.69	1.33	0.70	0.77	1.13
22	0.45	0.50	0.45	0.45	0.47	0.37	0.47	0.36
23	0.69	0.79	0.78	0.98	0.85	0.73	0.83	0.69
24	0.74	0.63	0.68	0.52	0.73	0.54	0.53	0.40

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APPENDIX E

Expectation of Mean Squares and Appropriate Error Terms for the Analyses of Variance of Tables 5, 6, and 7*

Source	Expectation of Mean Square	Error Term
1. Order (O)	$\sigma_e^2 + nms\ \sigma_o^2 + nm\ \sigma_{os}^2$	5
2. Modality (M)	$\sigma_e^2 + nos\ \sigma_m^2 = ns\ \sigma_{om}^2 +$ no $\sigma_{ms}^2 + n\sigma_{oms}^2$	(4 + 6)**
3. Subjects (S)	$\sigma_e^2 + nom\ \sigma_s^2 + nm\ \sigma_{os}^2$	5
4. O X M	$\sigma_e^2 + ns\ \sigma_{om}^2 + n\sigma_{oms}^2$	7
5. O X S	$\sigma_e^2 + nm\ \sigma_{os}^2$	7**
6. M X S	$\sigma_e^2 + no\ \sigma_{ms}^2 + n\ \sigma_{oms}^2$	7
7. O X M X S	$\sigma_e^2 + n\ \sigma_{oms}^2$	--

* o levels of O
m levels of M
s levels of S

** These error terms although not wholly appropriate were used as the best available in the design.

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APPENDIX F

Group Scale Values

APPENDIX F1

Group Log Scale Values

Condition	Stimuli 1	2	3	4	5	6	7
Circle size	0.1342	0.4492	0.8263	1.0303	1.1818	1.3146	1.4686
Numerosity	0.9753	1.1902	1.3247	1.4397	1.5634	1.6661	1.7796
Handgrip	0.7009	0.8258	1.0588	1.3244	1.5596	1.7008	1.8408
CMM Circle Size	0.7251	0.9613	1.1612	1.3726	1.4638	1.6145	1.7601
CMM Numerosity	0.8976	1.1276	1.2572	1.3861	1.4805	1.6228	1.7642

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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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APPENDIX F2

Group Order-1 and Order -2 Log Scale Values

Condition	Stimuli 1	2	3	4	5	6	7
Circle Size							
0-1	0.1486	0.4537	0.8514	1.0510	1.2062	1.3294	1.4776
0-2	0.1199	0.4448	0.8012	1.0095	1.1575	1.2999	1.4595
Numerosity							
0-1	0.9884	1.2030	1.3372	1.4404	1.5710	1.6768	1.7726
0-2	0.9621	1.1777	1.3123	1.4390	1.5558	1.6555	1.7867
Force of Handgrip							
0-1	0.7035	0.8116	1.0315	1.3308	1.5704	1.6944	1.8394
0-2	0.6993	0.8296	1.0860	1.3180	1.5487	1.7103	1.8425
CMM Circle Size							
0-1	0.7619	0.9755	1.1831	1.3796	1.4809	1.6046	1.7673
0-2	0.6883	0.9472	1.1393	1.3657	1.4466	1.6245	1.7528
CMM Numerosity							
0-1	0.9322	1.1602	1.2866	1.4012	1.5035	1.6284	1.7737
0-2	0.8630	1.1071	1.2277	1.3710	1.4574	1.6173	1.7546

APPENDIX F3

Group Log Scale Values for each Trial

Condition	Stimuli 1	2	3	4	5	6	7
Circle Size							
Trial 1	0.2280	0.4810	0.8275	1.0114	1.1931	1.3145	1.4564
2	0.1293	0.4176	0.8563	1.0569	1.2020	1.3291	1.4806
3	0.1110	0.4802	0.8463	1.0895	1.2065	1.3813	1.5027
4	0.1195	0.4360	0.8754	1.0462	1.2232	1.2928	1.4706
5	0.1271	0.4214	0.7896	1.0158	1.1421	1.2945	1.4568
6	0.1194	0.4604	0.7960	1.0042	1.1274	1.3076	1.4748
7	0.1264	0.4433	0.8032	1.0001	1.1947	1.3039	1.4720
8	0.1065	0.4391	0.8162	1.0180	1.1656	1.2936	1.4564
Numerosity							
Trial 1	1.0174	1.1758	1.3354	1.4570	1.5318	1.6930	1.7088
2	0.9869	1.2143	1.3372	1.4360	1.5895	1.6504	1.8088
3	0.9809	1.2095	1.3279	1.4307	1.5805	1.7098	1.8041
4	0.9685	1.2124	1.3485	1.4405	1.5823	1.6539	1.7686
5	0.9552	1.1430	1.2599	1.4105	1.5376	1.6542	1.7846
6	0.9610	1.1884	1.3255	1.4486	1.5672	1.6884	1.7850
7	0.9698	1.1830	1.3399	1.4321	1.5610	1.6344	1.7863
8	0.9629	1.1963	1.3237	1.4648	1.5599	1.6450	1.7909
Force of Handgrip							
Trial 1	0.7075	0.7904	1.0007	1.3211	1.6553	1.7731	1.8696
2	0.6914	0.8085	0.9918	1.3391	1.4863	1.6990	1.8363
3	0.6878	0.8309	1.0478	1.3271	1.5642	1.6817	1.8257
4	0.7271	0.8166	1.0900	1.3361	1.5756	1.6237	1.8262
5	0.6677	0.7768	1.1532	1.3800	1.5686	1.7476	1.8535
6	0.7231	0.8202	1.0668	1.2765	1.5633	1.7213	1.8280
7	0.6825	0.8364	1.0345	1.2929	1.5399	1.6908	1.8555
8	0.7191	0.8848	1.0895	1.3227	1.5231	1.6800	1.8329

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Group Log Scale Values for each Trial (Cont'd.)

Condition	Stimuli 1	2	3	4	5	6	7
CMM Circle Size							
Trial 1	0.7967	1.0045	1.1508	1.4528	1.5318	1.5868	1.7493
2	0.7877	0.9670	1.1803	1.3803	1.4521	1.6286	1.7628
3	0.7125	0.9603	1.1545	1.3932	1.4879	1.5840	1.7923
4	0.7506	0.9749	1.2468	1.3759	1.4490	1.5355	1.7648
5	0.7072	0.9613	1.0862	1.4321	1.4412	1.6292	1.7217
6	0.7084	0.9229	1.1060	1.3297	1.3900	1.6127	1.7578
7	0.6356	0.9224	1.1695	1.3147	1.4669	1.6271	1.7676
8	0.7018	0.9783	1.1956	1.3449	1.4883	1.6295	1.7627
CMM Numerosity							
Table 1	0.9422	1.2170	1.2766	1.3964	1.5640	1.6514	1.7577
2	0.9750	1.1762	1.3064	1.4151	1.5027	1.6217	1.7622
3	0.9346	1.1124	1.2629	1.3815	1.5024	1.6088	1.7954
4	0.9074	1.1534	1.2964	1.4120	1.4552	1.6231	1.7654
5	0.8084	1.0546	1.1903	1.4501	1.4694	1.6832	1.7401
6	0.9109	1.1667	1.2625	1.3492	1.4912	1.5914	1.7278
7	0.8794	1.0901	1.2011	1.3620	1.4186	1.5924	1.7802
8	0.8409	1.1170	1.2591	1.3268	1.4462	1.5999	1.7712